1966-05

A study of digital control design methods.

Steele, Harry A. Jr.

Monterey, California: U.S. Naval Postgraduate School

http://hdl.handle.net/10945/28571

A STUDY OF DIGITAL CONTROL DESIGN METHODS

HARRY A. STEELE

A STUDY OF DIGITAL

CONTROL DESIGN METHODS

by

Harry A. Steele Jr.
Lieutenant, United States Navy
B.S., South Dakota State College, 1958

Submitted in partial fulfillment
for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

UNITED STATES NAVAL POSTGRADUATE SCHOOL
May 1966

ABSTRACT

The development of microelectronics has brought into being large capacity digital memories in a small package. In the foreseeable future even more advances can be seen in this trend. Therefore the use of digital computers in control systems will play an even larger role than today.

This work involves a fourth order system to simulate the control and dynamics of a missile. Proportional navigation is used as the guidance method. Studied are the effects of applying different controls which are considered best from a computer study and the effects of applying digital filtering methods. Although these studies were applied to a specific problem, an attempt is made to keep the discussion general in order that the methods may be considered for other problems.

TABLE OF CONTENTS

Section	Title	Page
Chapter I	Introduction	11
Chapter II	Proportional Navigation	14
Chapter III	Filter and Control	21
Chapter IV	Digital Simulation	31
Chapter V	Results from Simulation	41
	Bibliography	70
Appendix I	Program Listings	72
Appendix II		88

LIST OF TABLES

Table		Page
5-1	Simulation initial conditions	44

LIST OF ILLUSTRATIONS

Figure		Page
1-1	Block Diagram of a Dynamic System	12
2-1	Constant Bearing Trajectory	16
2-2	Proportional Navigation Geometry	16
2-3	Missile Guidance Inter Loop	17
2-4	Missile Guidance Outer Loop	17
3-1	Flow Diagram of the System	22
4-1	Block Diagram of Missile Simulation	32
4-2	Discrete Flow Graph of the Plant	33
4-3	Discrete Flow Graph of the Filter/ Predictor	35
4-4	Geometry of the Kinematics	38
4-5	Block Diagram of the Kinematics	38
4-6	Discrete Flow Graph of Missile Simulation	40
5-1	Initial Condition of Target	42
5-2	Target Trajectories	43
5-3 to 5-26	Target Trajectories from Simulation	46-69

LIST OF SYMBOLS

γ	Missile flight path angle
k	Navigation constant
LOS	Line-of-sight
σ	Line-of-sight angle
β	Difference between sigma and gamma
V_m	Speed of missile
V_t	Speed of target
S	Laplace operator
R	Range, missile to target
$\frown$	Effective navigation constant
X	State vector
U	Control vector
F	System matrix
D	Control system matrix
b	Forward gains
a	Feedback gains
B	Feed forward gain vector
Φ	Transition matrix
Δ-DEL	Distribution matrix
J	Cost function
Q	Weighting matrix
r	Weighting constant
AT	Feedback vector
f(x)	Density function
F(X)	Density function matrix
P	Covariance error matrix

Z	Noisy observable vector
H	Observable matrix
V	Measurement noise vector
R	Covariance matrix of measurement noise
σx	Standard deviation
L	Loss function
∇	Gradiant
$\hat{X}$	Best estimate vector based on past information
X*	Best estimate vector based on present information and past prediction
G	Optimum gain matrix
W	Excitation noise vector
Q	Covariance matrix of excitation noise
DI	DINP-Deterministic input

CHAPTER I

INTRODUCTION

If it is desired to hit a target with a missile, some form of guidance must be employed. This guidance would be selected weighing the desired objectives and an estimate of the target capabilities against the amount of money and missile space available. As suggested in the introduction, the advances in computer size and capabilities is, and will in the future, continue to change the weighting of these factors. Thus in the foreseeable future it may be possible to design a model of the system which will be in sufficient detail to predict the response of the real system to a high degree of accuracy. Using the micro-computers, this model could then become a part of the control system in the missile. Such a control system might be described in the block diagram in Figure 1-1.

In Figure 1-1 the missile measures and reacts to a signal which depends on the type of guidance employed. Such a signal might depend on the rate of change in the missile's radar seeker head which is tracking the line of sight from the missile to target. The missile will react to this signal, producing through its dynamics a change in the direction of its velocity vector, or the missile will take some other action called for by the particular guidance law being used. The model receives the measured signals and considers the measurable states from the missile dynamics. Since all the states of the model are measurable and the digital model in itself generates relatively little noise, the model becomes a

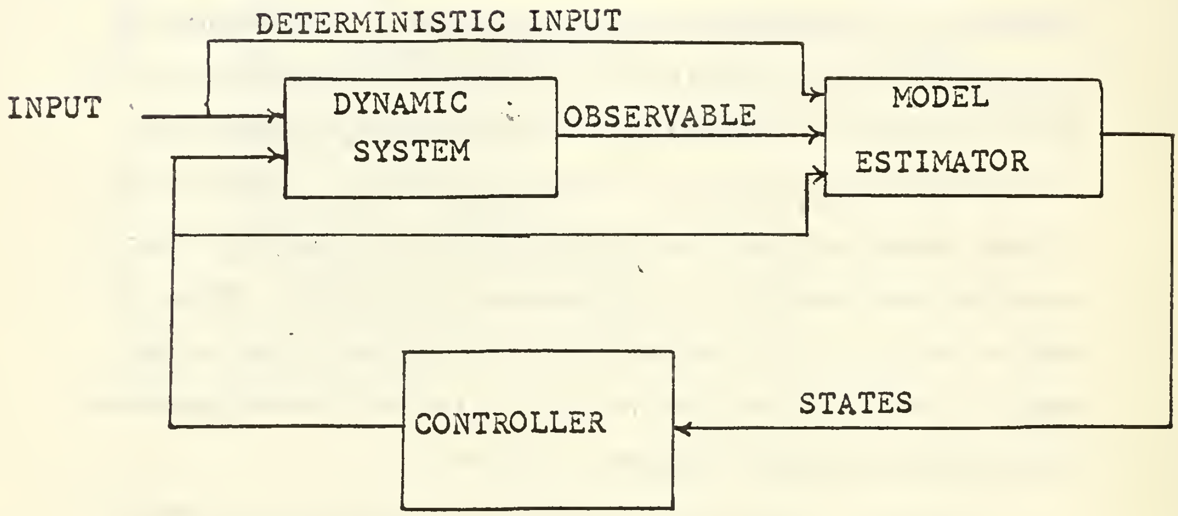


Figure 1-1. Block Diagram of a Dynamic System.

state predictor or estimator.

Since all the states of the model are available, a control law considering all the states can now be designed. This control law is applied in the controller to guide the missile.

If the desired objectives of the system can justify the above approach, the system performance of the modified system should be an improvement over most conventional guidance systems.

CHAPTER II

PROPORTIONAL NAVIGATION

Adler in [9] defined proportional navigation as,

> A course in which the rate of change of missile heading is directly proportional to the rate of rotation of the line-of-sight from the missile to the target.

$$\dot{\gamma} = K \dot{\sigma} \tag{1}$$

where

$\dot{\gamma}$ is the angular rate of change of the missile velocity vector.

$\dot{\sigma}$ is the angular rate of change of the line-of-sight.

K is a constant, typically between three and five.

To gain a better understanding of proportional navigation, some of the simpler forms of line-of-sight guidance will be examined. Pursuit course (sometimes called pure pursuit) always aims the missile directly at the target along the line-of-sight. This method will always end in a tail chase, even if the missile is launched head-on with the target. High missile accelerations are required. It can be shown from the equations of motion that if the missile to target speed ration exceeds two, the final missile turning rate will approach infinity. Thus pursuit course guidance may not be very satisfactory although it is very simple to implement.

The next step is to lead the line-of-sight by an angle which is a function of the target velocity. This type is called constant bearing course, and is achieved by aligning the relative missile to target velocity vector with the line-of-sight. That is to say the line-of-sight maintains a constant direction in space. It can be shown that this method can cause a missile to follow a target to a collision even if the target is maneuvering. This method however requires an instantaneous correction to each change in the line-of-sight.

Consider the constant bearing trajectory shown in Figure 2-1. In the figure the line-of-sight is shown for several samples made by the radar. As long as the missile remains on the collision course, the angular rate of change of the line-of-sight will remain zero and no control action is required. Any rotation of the line-of-sight, indicating a departure from the collision course will be detected by the missile's radar. The missile using proportional navigation will turn at a rate proportional to this rotation in a direction to reduce the line-of-sight rate and return to a constant bearing collision course.

In the Figure 2-2 above, the lead angle β is defined:

$$\beta = \sigma - \gamma \tag{2}$$

This angle (β) is independent of any reference used to define σ and γ. If, as in the discussion above, we could achieve a constant bearing course, β would remain a constant, and therefore $\dot{\beta}$ would remain zero.

Thus for a constant bearing course,

$$\dot{\beta} = \dot{\sigma} - \dot{\gamma} = 0 \tag{3}$$

Since the missile cannot react instantaneously, $\dot{\beta}$ is not always zero. Thus for any instant of time (1) may be written as,

$$K\dot{\sigma} = \dot{\beta} + \dot{\gamma} \tag{4}$$

If $\dot{\beta}$ is equal to zero, the missile is on a constant bearing collision course. Therefore the object of the control system

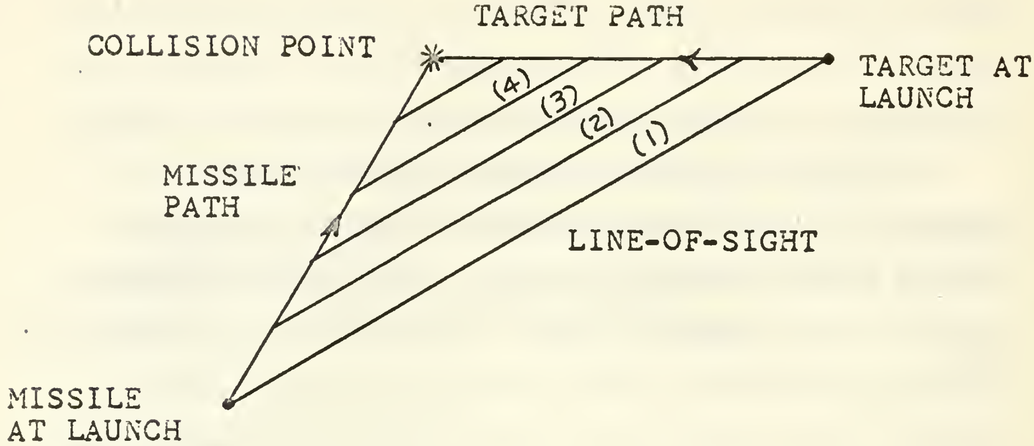


Figure 2-1. Constant Bearing Trajectory.

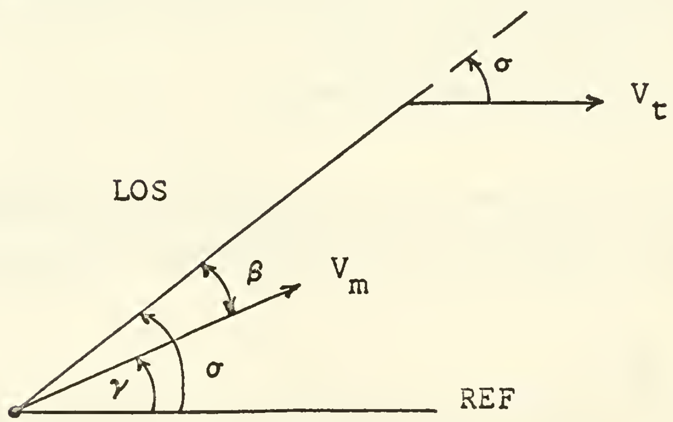


Figure 2-2. Proportional Navigation Geometry.

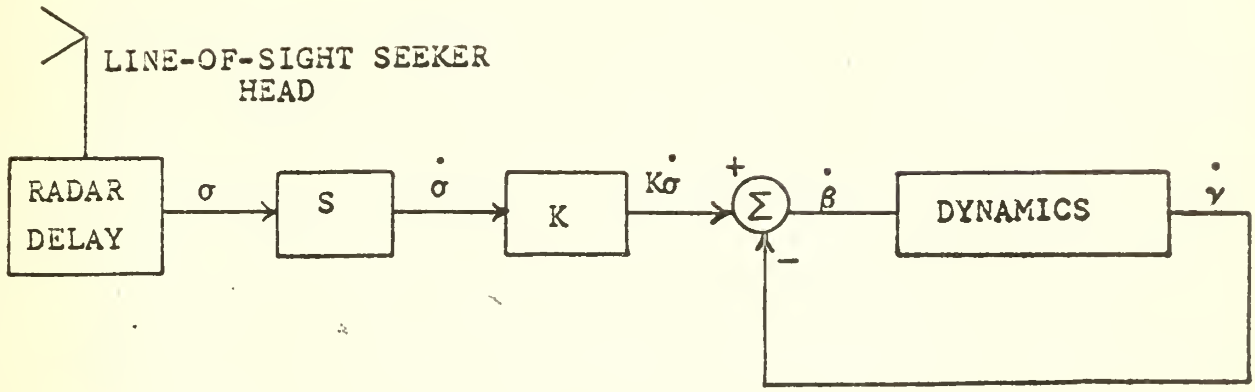


Figure 2-3. Missile Guidance Inter Loop.

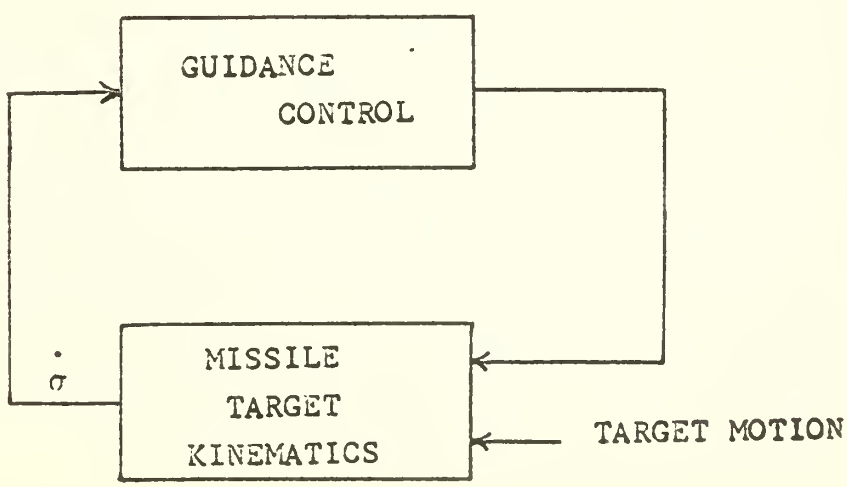


Figure 2-4. Missile Guidance Outer Loop.

in proportional navigation is to minimize $\dot{\beta}$ and if $\dot{\sigma}$ becomes a constant or zero to drive $\dot{\beta}$ to zero.

Guidance and Control

A typical homing missile inter guidance loop is shown in Figure 2-3. The following discussion will be limited to only coplanar motion of the missile and target. A constant missile and target speed is also assumed throughout the rest of this paper.

In Figure 2-3, the radar tracks the target using a servo loop to keep the antenna on the line-of-sight. A delayed signal which is a measure of σ, is obtained from the radar system. A voltage which is proportional to the rate of change of this signal is obtained. This voltage is compared with the output of the dynamic system $\dot{\gamma}$, and the error signal is obtained. This error signal is sent either to the hydraulic system to rotate the control surfaces or to a hydraulic/valve system which controls side thrust jets, depending on the type of missile being used.

Kinematics

As might be suggested by the name inter loop, there is also an outer loop as shown in Figure 2-4. In the Figure, the guidance and control block is shown as in Figure 2-3. The effect of $\dot{\gamma}$ on the line-of-sight and the effect of the target motion on the line-of-sight are represented in the missile-target kinematics block.

This outer loop is not as accessible to measurement and control, but the understanding of its effects is necessary in order to perform a simulation of the entire system. From the geometry of Figure 2-2, the following equations may be written:

$$\dot{R} = V_t \ COS(\sigma) - V_m \ COS(\beta) \tag{5}$$

$$R\dot{\sigma} = -V_t \ SIN(\sigma) + V_m \ SIN(\beta) \tag{6}$$

where

V_t = Target speed (assumed Constant)

V_m = Missile speed (assumed Constant)

$\beta = \sigma - \gamma$

R = the line-of-sight distance

$\dot{R}$ = the rate of change of the line-of-sight.

Then equations (5) and (6) may be rewritten:

$$\dot{R} = V_t \ COS(\sigma) - V_m \ COS(\sigma - \gamma) \tag{7}$$

$$R\dot{\sigma} = -V_t \ SIN(\sigma) + V_m \ SIN(\sigma - \gamma) \tag{8}$$

Differentiating (8) with respect to time:

$$\dot{R}\dot{\sigma} + R\ddot{\sigma} = -V_t COS(\sigma)\ \dot{\sigma} + V_m \ COS(\sigma - \gamma)(\dot{\sigma} - \dot{\gamma})$$

$$= -\dot{R}\dot{\sigma} - V_m COS(\sigma - \dot{\gamma})\ \dot{\gamma} \tag{9}$$

Therefore

$$2\dot{R}\dot{\sigma} + R\ddot{\sigma} = -V_m \ COS(\sigma - \gamma)\ \dot{\gamma} \tag{10}$$

let

$$\dot{R} = -R/T$$

and

$$V_r = |\dot{R}|$$

where T is the time to go to the target. Then equation (10) may be represented by:

$$(-2R/T + RS)\ \dot{\sigma} = -V_m \ COS(\sigma - \gamma)\ \dot{\gamma} \tag{11}$$

thus

$$\dot{\sigma}/\dot{\gamma} = \frac{-V_m \ COS(\sigma - \gamma)}{R/T \ (TS - 2)} \tag{12}$$

Where $\dfrac{V_m K \ COS(\sigma - \gamma)}{R/T}$ is called the effective navigation constant.

Since V_m is constant and variations in $\cos(\sigma - \gamma)$ and V_r are small, this term is assumed constant and values of four to six are considered best.

CHAPTER III

FILTER AND CONTROL

Plant Description

A linear, time-invariant dynamic system is described in the flow graph, Figure 3-1.

The transfer function of Figure 3-1 may be stated:

$$\frac{X_{out}(s)}{X_{in}(s)} = \frac{b_n s^{n-1} + \ldots\ldots + b_2 s + b_1}{s^n + a_n s^{n-1} + \ldots + a_2 s + a_1} \tag{1}$$

Equation (1) may be written in one differential equation using matrix notation.

$$\dot{X} = F\,X + D\,U, \qquad C = B\,X \tag{2}$$

Where

X is the state vector (n x 1)

U is a scalar of the system inputs and controls

F is a matrix of constants. For the system described in Figure 3-1, F is defined in equation (3).

$$F = \begin{bmatrix} 0 & 1 & 0 & . & . & 0 \\ 0 & 0 & 1 & . & . & 0 \\ . & . & . & . & . & . \\ 0 & 0 & 0 & . & 1 & 0 \\ 0 & 0 & 0 & . & 0 & 1 \\ -a_1 & -a_2 & . & . & . & -a_n \end{bmatrix} \tag{3}$$

D is a matrix of constants (n x 1) described in equation (4) for the system in Figure 3-1.

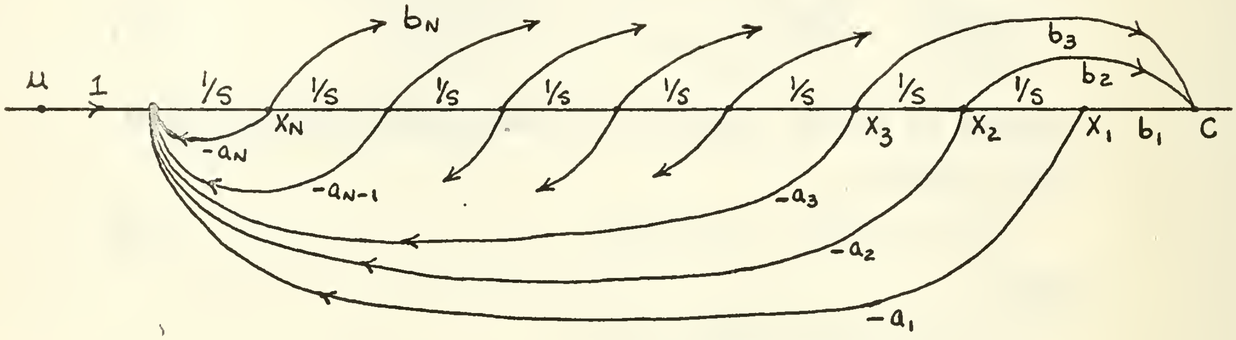


Figure 3-1. Flow Diagram of the system.

$$D = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ 0 \\ 1 \end{bmatrix} \tag{4}$$

B is an (1 x n) row vector, described in equation (5) for the system described in Figure 3-1.

$$B = [b_1 \; b_2 \; . \; . \; . \; b_n] \tag{5}$$

The solution to (2) is given by:

$$X(t) = \Phi(t - t_o) X(t_o) + \int_{to}^{t} \Phi(t - t_1) D \, dt_1 \, U(t_o)^{1} \tag{6}$$

Where

$$\Phi(t - t_o) = \mathcal{L}^{-1} (SI - F)^{-1}$$

Solving the differential equation (2) for the discrete solution,

$$X(k + 1) = \Phi X(k) + DEL \, U(k) \tag{7}$$

Where

$$\Phi = e^{F(t - t_o)} \tag{8}$$

$$DEL = \int_{to}^{t} e^{F(t - t_1)} D \, dt_1 \tag{9}$$

Note that $k(t - t_o)$ is represented by k.

Titus, in reference [10] developed a digital computer program called PHIDEL. This program provides a solution to equations (8) and (9) and will be used as a subroutine to obtain Φ and DEL. A listing of the PHIDEL program is provided in Appendix one.

Plant Control

It is desired to design a set of feedback coefficients to control the plant in accordance with a selected performance

[1] U(t) is held constant over the interval $(t - t_o)$ and is equal to $U(t_o)$.

index. This set of coefficients will modify the plant states and in effect will cause a movement of the Z-plane poles. This effect must be weighed against the feedback gains derived for the desired performance index.

Let J(N) be defined as the performance index of a discrete sample-data system which will be minimized with respect to U(K), the control for the plant.

$$J(N) = \min_{U(K)} \sum_{K=M}^{N} [X^t(K)\, Q\, X(K) + r\, U^2(K-1)] \tag{10}$$

Q is a (n x n) positive-definite symmetric constant matrix, and R is a positive scalar constant.

The principle of optimality states that any portion of an optimal trajectory is also an optimal trajectory. Therefore equation (1) may be rewritten:

$$J(N) = \min_{U(N)} [X^t(N)\, Q\, X(N) + r\, U^2(N-1) + J(N-1)] \tag{11}$$

If the gradient of J(N) is taken with respect to U(N - 1) and set equal to zero, then an optimal control, $U^o(N-1)$ may be determined. It is noted that J(N - 1) is not a function of U(N - 1).

Using the above arguments, Titus developed the following algorithms:

$$A^t(k+1) = \frac{-\Delta^t P(k)\, \Phi}{\Delta^t P(k)\, \Delta + r} \tag{12}$$

$$P(k) = \Psi^t(k) P(k-1)\, \Psi(k) + Q + rA(k)A^t(k) \tag{13}$$

$$\Psi(k) = \Phi + \Delta A^t(k) \tag{14}$$

Using the recursive relations of (12), (13) and (14), Titus [10] developed program OPTCON. This method is also discussed in references [3] and [4] by Titus, Strum and Demetry, and in reference [6] by Ogden. A fortran listing of the program may be found in Appendix one.

Digital Filter

In the control discussion, the assumption was made that all the states of the dynamic system are observable states. This condition is certainly not always true and often the observable states may be measured only at the cost of some ambiguity due to the measurement noise. The inputs to the system such as the radar seeker discussed in Chapter Two will often be received in a noisy environment.

Kalman in references [1] and [2] discussed these problems and presented the theory for the desired filter. Schmidt [5] ; Titus, Demetry and Strum [4] and Jardine [7] have discussed this problem and developed methods of implementing this problem on the digital computer. In an effort to maintain clarity, some of these developments will be presented.

The digital filter will provide a best estimate of all the states by weighing the past information with the present observable states. This weighting is performed with the knowledge of the environment (excitation) and the measurement noise. Although in many cases the noise is very difficult to describe, the assumption that white noise is present is in general true. Thus if white noise can be discriminated, the integrity of the measured signals will be improved.

Filter Design

For a single variable, the desnity function f(x) for a gaussian distribution is given by:

$$f(x) = \frac{1}{(2\pi)^{\frac{1}{2}} \sigma} e^{-\frac{1}{2}\frac{(x-u)^2}{\sigma^2}} \tag{15}$$

where

$$\int g(x) f(x) dx = E[g(x)] \tag{16}$$

where E implies expected value,

and

$$\int f(x) dx = 1 \tag{17}$$

Assume that the systems measurement and excitation noise are white noise, where white noise has a gaussian distribution and is spread uniformly over all frequencies.

Noting that the input, U(k) to the plant is independent of U(k - 1), X(k) is independent of X(k - 1) and using the joint probability property of random independent samples, a density function for the state system may be written:

$$F(X) = \frac{1}{(2\pi)^{n/2} P^{\frac{1}{2}}} e^{-\frac{1}{2}[X - \hat{X}]^t P^{-1} [X - \hat{X}]} \tag{18}$$

where $\hat{X}(k)$ is the best estimate of the states based on past information.

$$Z(k-1) = H X(k-1) + V(k-1) \tag{19}$$

Z is the noisy observable matrix, and V is the measurement noise vector defined by:

$$E[V] = 0 \tag{20}$$

$$E[VV^t] = R \tag{21}$$

R is the covariance matrix of the measurement noise and P is the covariance matrix of the error. P is a symmetric matrix, $(P_{ij} = P_{ji})$. The diagonal terms of P are equal to the variance of each state $(\sigma^2_{x_i})$ and the off-diagonal terms are equal to correlation between the states,

$(E[x_i \cdot x_j] - \hat{x}_i \cdot \hat{x}_j)$.

$$P = E[[X - \hat{X}] \cdot [X - \hat{X}]^t] \tag{22}$$

The expected value of the error is defined as the loss function:

$$L = \int (X - \hat{X})^t (X - \hat{X}) F(X/Z,\hat{X}) dX \tag{23}$$

Taking the gradiant of equation (23) with respect to $\hat{X}$;

$$\nabla_{\hat{X}} L = \int 2XF(X/Z,\hat{X})dX - 2\hat{X} \int F(X/Z,\hat{X})dX \tag{24}$$

Applying equations (16) and (17) and setting the result equal to zero

$$\nabla_{\hat{X}} L = 2 \quad E \quad X/Z,\hat{X} \; - \; 2\,\hat{X} = 0$$

Thus:

$$X^* = \mathrm{Max}(\hat{X}) = E[\,X/Z,\hat{X}\,] \tag{25}$$

X^* is the best estimate of the states given the present noisy observable and the past prediction of the states.

Filter Equations

There are two methods of finding the recursive relations for the filter program. Method one assumes the random varables are gaussian and makes use of Bayes equation to derive an expression for equation (25). Method two assumes a linear filter, selects an algorithm for equation (25) and proves that this selection provides the best values for X^*.

Method One

The covariance matrix of the observable error may be written:

$$P_z = E[(Z - \hat{Z})(Z - \hat{Z})^t] \tag{26}$$

Combining equations (19) and (26),

$$\begin{aligned} P_z &= E\,[\,(HX + V - H\hat{X})(HX + V - H\hat{X})^t] \\ &= H\,E[(X - \hat{X})(X - \hat{X})^t\,] + E[VV^t\,] \end{aligned}$$

Using equations (20), (21) and (22),

$$P_z = HPH^t + R \tag{27}$$

In order to find an expression for X*, Bayes formula is applied to equation (25).

$$X^* = F(X/Z,\hat{X}) = \frac{F(Z,\hat{X}/X)\ F(X)}{F(Z,\hat{X})} \tag{28}$$

Since Z(k) is independent of $\hat{X}(k)$, equation (28) may be written:

$$F(X/Z,\hat{X}) = \frac{F(Z/X)\ F(\hat{X}/X)\ F(X)}{F(Z)\ F(\hat{X})} \tag{29}$$

Since $F(\hat{X}/X) = \dfrac{F(X/\hat{X})\ F(\hat{X})}{F(X)}$, equation (29) becomes,

$$F(X/Z,\hat{X}) = \frac{F(Z/X)\ F(X/\hat{X})}{F(Z)} \tag{30}$$

Since X(k) is independent of $\hat{X}(k)$, (30) becomes,

$$F(X/Z,\hat{X}) = \frac{F(Z/X)\ F(X)}{F(Z)} \tag{31}$$

Following the form in equation (18), F(Z) and F(Z/X) are defined:

$$F(Z) = \frac{1}{(2\pi)^{n/2}|P_Z|^{\frac{1}{2}}}\ e^{-\frac{1}{2}(Z - \hat{Z})^t P_Z{}^{-1}(Z - \hat{Z})} \tag{32}$$

$$F(Z/X) = F(V) = \frac{1}{(2\pi)^{n/2}\ |R|^{\frac{1}{2}}}\ e^{-\frac{1}{2}(V^t R^{-1} V)} \tag{33}$$

Combining equations (18), (31) and (33),

$$F(X/Z,\hat{X}) = A\ e^{-\frac{1}{2}B}$$

Where

$$A = \frac{(HPH^t + R)^{\frac{1}{2}}}{(2\pi)^{n/2}\ |R|^{\frac{1}{2}}\ |P|^{\frac{1}{2}}}$$

and

$$B = V^t R^{-1} V - (X - \hat{X})^t P^{-1}(X - \hat{X}) + (Z - \hat{Z})^t (HPH^t + R)^{-1}(Z - \hat{Z})$$

Setting the gradiant of B with respect to X equal to zero.

$$\nabla_X B = \tfrac{1}{2}(2(X^* - \hat{X})^t P^{-1} - 2(Z-\hat{Z})^t(HPH^t+R)^{-1}\nabla_X(Z-\hat{Z}) = 0$$

since

$\nabla_X(Z-\hat{Z}) = \nabla_X(HX + V) = H$, the algorithm, equation (35) may be written.

$$X^* = \hat{X} + PH^t(HPH^t + R)^{-1}(Z - \hat{Z}) \tag{35}$$

Let the filter gain matrix G be defined.

$$G = PH^t(HPH^t + R)^{-1} \tag{36}$$

Then the algorithm, equation (35) may be written:

$$X^* = \hat{X} + G(Z - \hat{Z}) \tag{37}$$

Method Two

An alternate method of deriving the recursive relation for the gain matrix assumes a linear filter. The trace of the covariance matrix is given by:

$$L = E[(X - \hat{X})^t(X - \hat{X})] \tag{38}$$

It will be shown that if L is minimized with respect to the gain, then the gain will be given by equation (36).

Substituting equation (37) into equation (38),

$$L = E[(X-\hat{X})(X-\hat{X})^t] - E[(X-\hat{X})(Z-\hat{Z})^tG^t] - E[G(Z-\hat{Z})(X-\hat{X})^t] + E[G(Z-\hat{Z})(Z-\hat{Z})^tG^t] \tag{39}$$

Since $Z = HX + V$

Then $\hat{Z} = E[HX + V] = H\hat{X}$

Assuming that $E[(X-\hat{X})V^t]$ is equal to zero, equation (39) may be written:

$$L = E[(X-\hat{X})(X-\hat{X})^t] - E[(X-\hat{X})(X-\hat{X})^t]H^tG^t + GE[VV^t] - GH\,E[(X-\hat{X})(X-\hat{X})^t] + GHE[(X-\hat{X})(X-\hat{X})^tH^tG^t \tag{40}$$

Substituting equations (21) and (22) into equation (40),

$$L = P - PH^tG^t - GHP + GHPH^tG^t + GRG^t \tag{41}$$

Taking the gradiant of L with respect to G and solving for G.

$$G = PH^t(HPH^t + R)^{-1}$$

Filter Programs

Jardine, Titus, Demetry and Strum have designed programs to calculate the filter gains and the covariance matrix of error.

Such a program, (Program Filter) is listed in Appendix one.

CHAPTER IV

DIGITAL SIMULATION

If a digital filter can be obtained, then a method must be developed to evaluate this system. This chapter will describe the simulation of the missile tracking system, the digital predictor, the control, and the kinematics. The results of this simulation for a particular missile will be discussed and trajectories from computer runs will be presented in Chapter Five.

Missile Guidance

Figure 4-1 is a block diagram of the digital simulation. As described in previous chapters, the missile system tracks the target, obtaining an input for the missile guidance. This signal is proportional to the angular rate of change of the line-of-sight angle. In this simulation it is assumed that white noise is added to this signal.

The details of the missile guidance simulation are described in Appendix Two. This portion of the simulation describes the tracking, control and dynamics of the missile as a transfer function relating the line-of-sight angular rate to the missile flight path angular rate. This transfer function is then formed into an F and D matrix as described in Chapter Three. Program Phidel is employed to obtain the Φ and Δ matrixes for the state difference equation.

The output of the missile guidance ($\dot{\gamma}$) is considered the observable for the predictor. Gaussian measurement noise with standard deviation (σ_V) is added to the observable. The output ($\dot{\gamma}$) is also an input to the kinematics. Figure 4-2

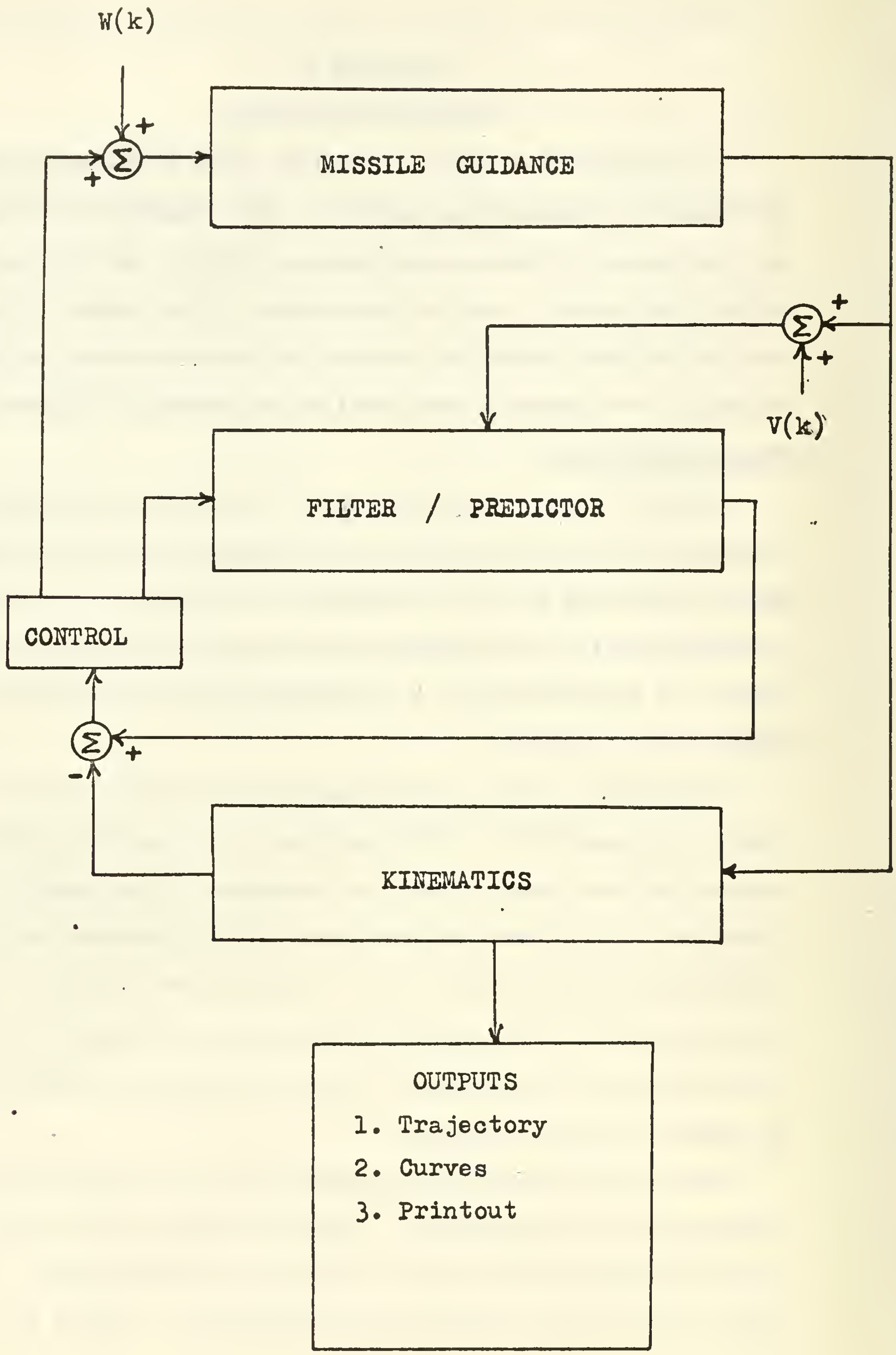


Figure 4-1. Block Diagram of Missile Simulation.

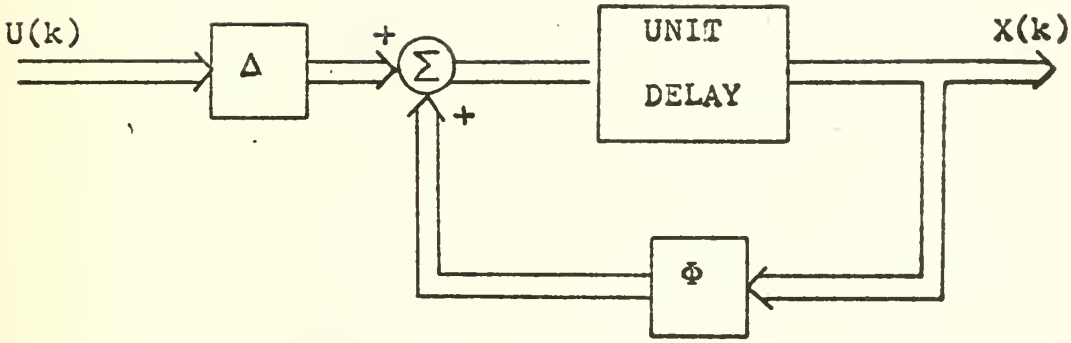


Figure 4-2. Discrete Flow Graph of the Plant.

describes the digital simulation of the guidance. The following fortran listing updates the discrete difference equation as demonstrated in Figure 4-2.

```
C    THIS SECTION UPDATES THE STATE VECTOR X
     CALL RNDEV(NUNIF,DEV)
     W=SIGW*DEV
     CALL PROD(PHI,X,KN,KN,1,TEMP1)
     DO 803 I = 1,KN
803  TEMP2(I,1) = W*DEL(I,1)
     CALL ADD(XS,DINP,KN,1,TELP)
     CALL PROD(AT,TELP,1,KN,1,TELP1)
     CALL PROD(DEL,TELP1,KN,1,1,TELP2)
     CALL ADD(TEMP1,TEMP2,KN,1,X)
     CALL ADD(X,TELP2,KN,1,X)
```

Digital filter-predictor

The digital filter is similar to the filter described in reference [4]. The gain matrix (G) is evaluated each sample instead of using the steady-state values of the gain. The discrete flow graph of the filter is shown in Figure 4-3.

One input to the filter is the noisy observable. This is the sum of the angular rate of the missile flight path ($\dot{\gamma}$) and the measurement noise. This noise is assumed gaussian with mean zero and variance σ_V. This measurement on the missile could be made by use of rate gyros.

The other input to the filter is the deterministic input. Here this would be the angular rate of the line-of-sight ($\dot{\sigma}$) and the best estimate of its derivatives. In the simulation

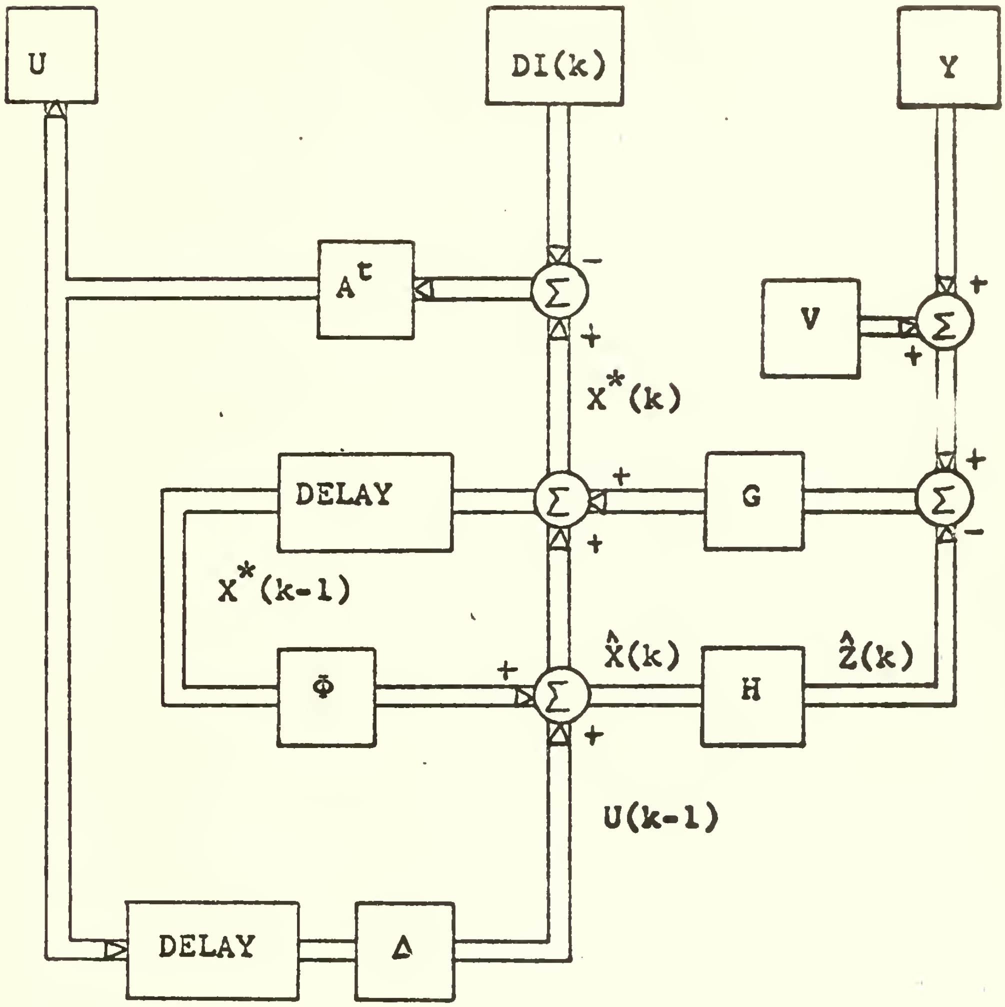


Figure 4-3. Discrete Flow Graph of the Filter/predictor

these are obtained from the kinematics. In the missile this signal would be measured by the tracking system. Note that the present value of the deterministic input DI(k) is used as the input to the plant while the past value of the input DI(k - 1) is used in the filter. The reasoning is that the filter is taking the present value of the observable Z(k) and therefore must use the past value of the deterministic input DI(k - 1) which caused the observable Z(k).

The fortran statements for the filter are listed below.

```
C     THIS SECTION CALCULATES XS, THE BEST ESTIMATE
C     OF THE STATE VECTOR
      CALL PROD(H,X,KP,KN,1,Y)
      DO 10 I=1,KP
      CALL RNDEV(NUNIF,DEV)
10    V(I,1) = SIGV(I)*DEV
      CALL ADD(Y,V,KP,1,Z)
      CALL PROD(PHI,XS,KN,KN,1,TEMP1)
      DO 11 I = 1,KN
      DO 11 J = 1,KN
11    TEMP2(I,J) = -TEMP2(I,J)
      CALL PROD(TEMP2,TEMP1,KN,KN,1,TEMP3)
      CALL ADD(TEMP1,TEMP3,KN,1,TEMP3)
      CALL ADD(XS,DINP,KN,1,TELP)
      CALL PROD(AT,TELP,1,KN,1,TEMP1)
      CALL PROD(DEL,TEMP1,KN,1,1,TELP)
      CALL PROD(TEMP2,TELP,KN,KN,1,TELP1)
      CALL ADD(TELP,TELP1,KN,1,TELP1)
      CALL PROD(G,Z,KN,KP,1,TELP2)
```

```
CALL ADD(TEMP3,TELP1,KN,1,XS)
CALL ADD(XS,TELP2,KN,1,XS)
```

The Kinematics

The kinematics combines the flight path angular rate of the missile, the missile speed (assumed constant) and the target's velocity vector to determine their effect on the line-of-sight angular rate ($\dot{\sigma}$) and the range rate ($\dot{R}$).

Figure 4-4 demonstrates the sign conventions used in this simulation. The trajectories which will be discussed later in Chapter Five use this same notation and sign convention.

Figure 4-5 demonstrates in block diagram form the kinematics. As noted in the figure, by-products of the kinematics are (1) the position of the missile and target, (2) the range and (3) the sign and magnitude of the range rate.

By addition of a few fortran statements, adjusting the target's velocity vector, the target can be maneuvered as a function of the missile trajectory in range (evasion) or on a predetermined course (attack).

The fortran statements for the kinematics are listed below.

```
C     THIS SECTION DETERMINES THE EFFECT OF THE
C     PLANT OF THE KINEMATICS
      GAMDOT = X(1,1)
      GAMMA = GAMMA+GAMDOT*DT
      XMDOT = VM*COSF(GAMMA)
      YMDOT = VM*SINF(GAMMA)
      YRDOT = YTDOT-YMDOT
      XRDOT = YTDOT-YMDOT
      RDOT = ((YRDOT*SINF(SIGMA))+(XRDOT*COSF(SIGMA)))
      RDSIG = ((YRDOT*COSF(SIGMA))-(XRDOT*SINF(SIGMA)))
```

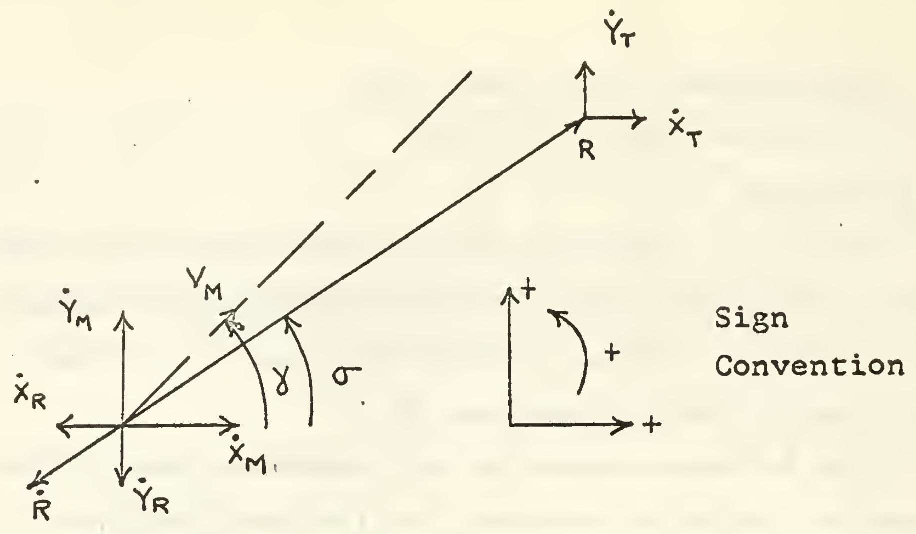


Figure 4-4. Geometry of the Kinematics.

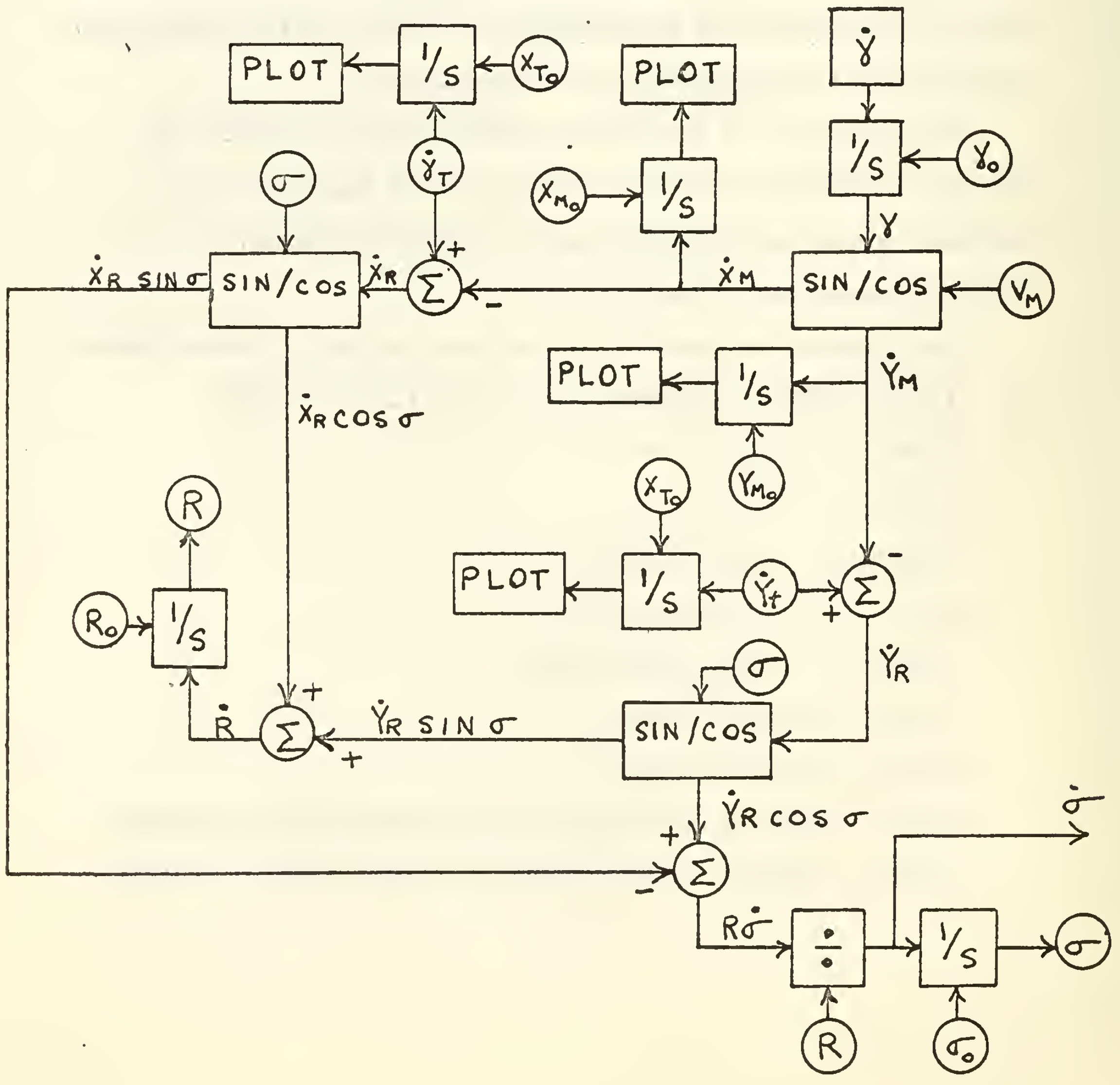


Figure 4-5. Block diagram of the Kinematics.

RLOS = RLOS+RDOT*DT

DSIG = RDSIG/RLOS

SIGMA = SIGMA+DSIG*DT

XMZ = XMZ+XMDOT*DT

YMZ = YMZ+YMDOT*DT

XTZ = XTZ+XTDOT*DT

YTZ = YTZ+YTDOT*DT

The Control

In this simulation a control (A^T) was selected through the use of program "OPCON" which minimizes the states X(k), ie, "bang-bang control". Other forms of control are also available such as placing a limit of the fuel or energy expended, placing a limit on the amount of acceleration allowed, or placing a limit on the magnitude of the angle and angular rate that can be measured. These options are recommended for future study and would, of course, be necessary in making a more complete study of the missile systems.

For this simulation the weighting vector (A^t) becomes a constant vector which is imposed on the difference between the best prediction of the states X*(k), and the deterministic input, DINP(k) to determine the control to be applied.

Figure 4-6 is a discrete flow graph of the entire simulation. The fortran listing of the entire program may be found in Appendix One.

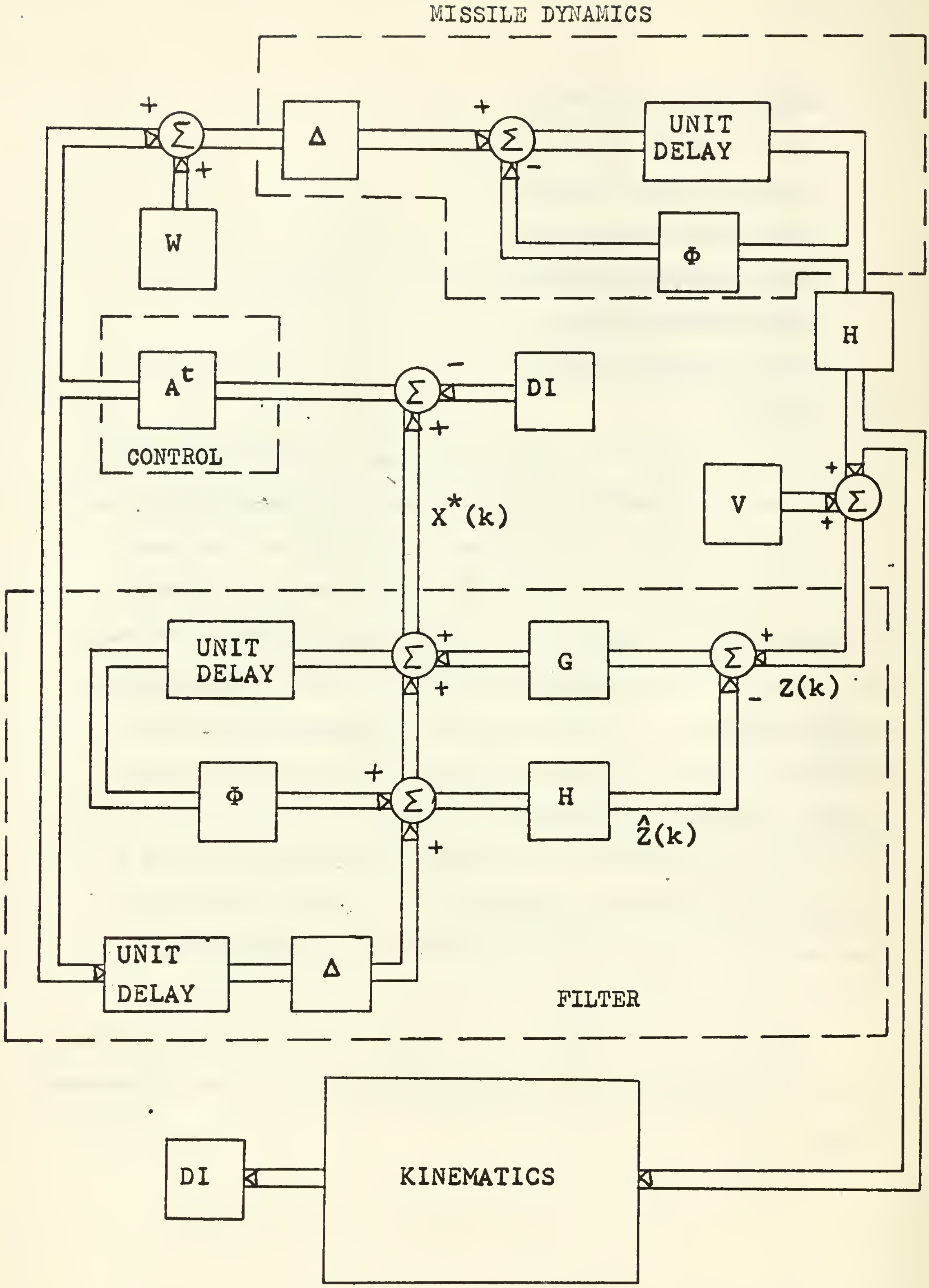


Figure 4-6. Discrete flow graph of missile simulation.

CHAPTER V

SIMULATION RESULTS

In making a study such as the evaluation of a missile guidance, it is quite difficult to decide which parameters to vary and which to hold constant. It is even more difficult to decide which curves hold the most meaning as a measure of success. Since the ultimate goal is to hit the target, trajectories and confirming print-out were used as a primary measure. Once a hit was obtained with a set of noise variances, the target trajectory was varied to see the effect on the missile. The noise was also varied to study the amount of noise that could be tolerated.

The measurement noise was found to be the most critical value. A standard deviation of measurement noise of 0.1 radians/sec was found to be the upper limit. Above this value the missile had some control, but the missile trajectories were far from desirable. Operation near the limit of measurement noise caused large ambiguities of the target information at the beginning of the flight. The target information improved as the range decreased, however, and in most cases the missile was able to capture the target.

Curves of the normal acceleration of the missile, the filter gains, the range, and the control versus time were considered and are shown in the results where these values appear to be critical or of interest.

This simulation assumed that the missile propulsion has already burned out and the missile is traveling at a constant velocity (V_m). This simulation considers only the coplanar situation. The program could be easily expanded to two channels

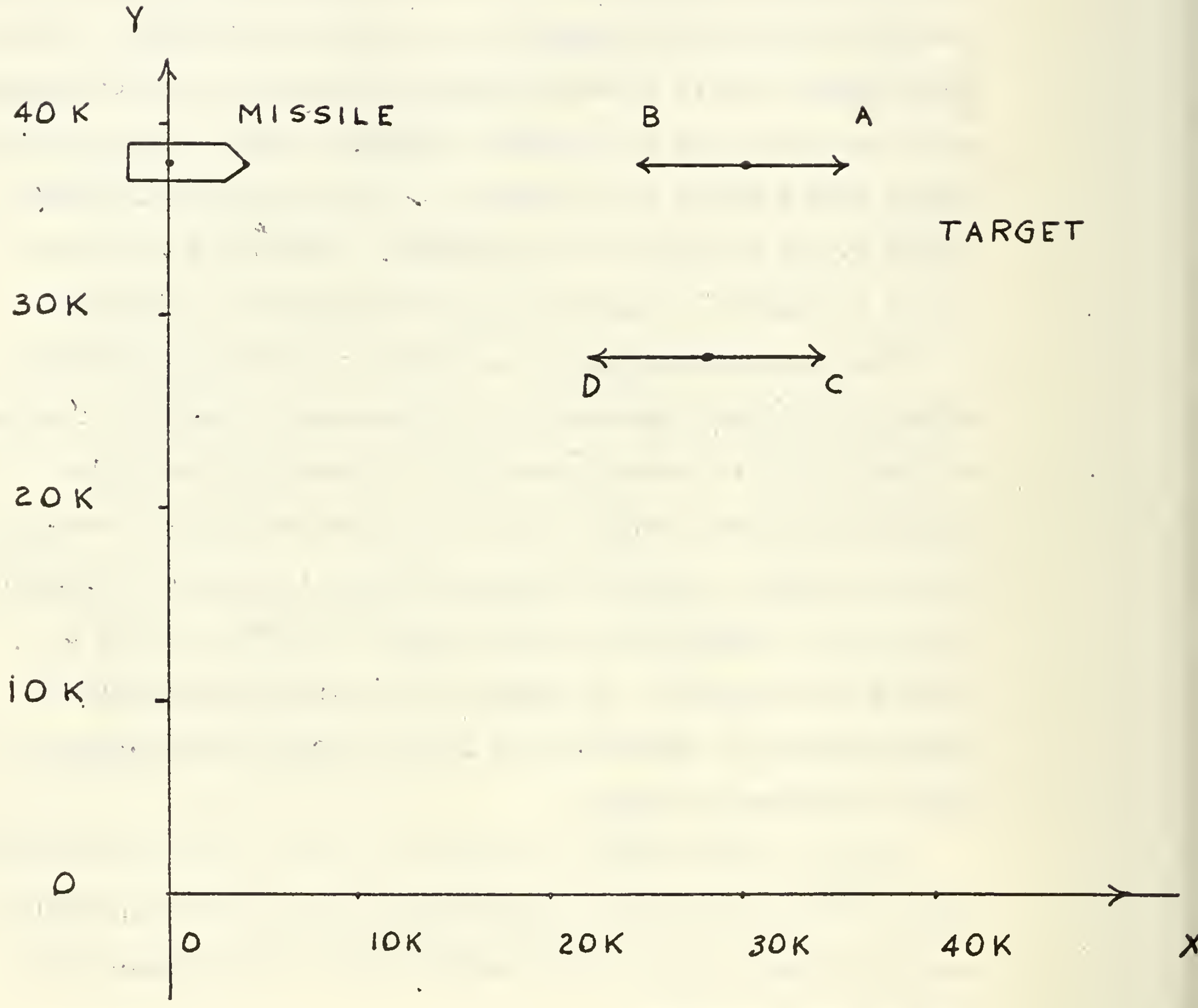
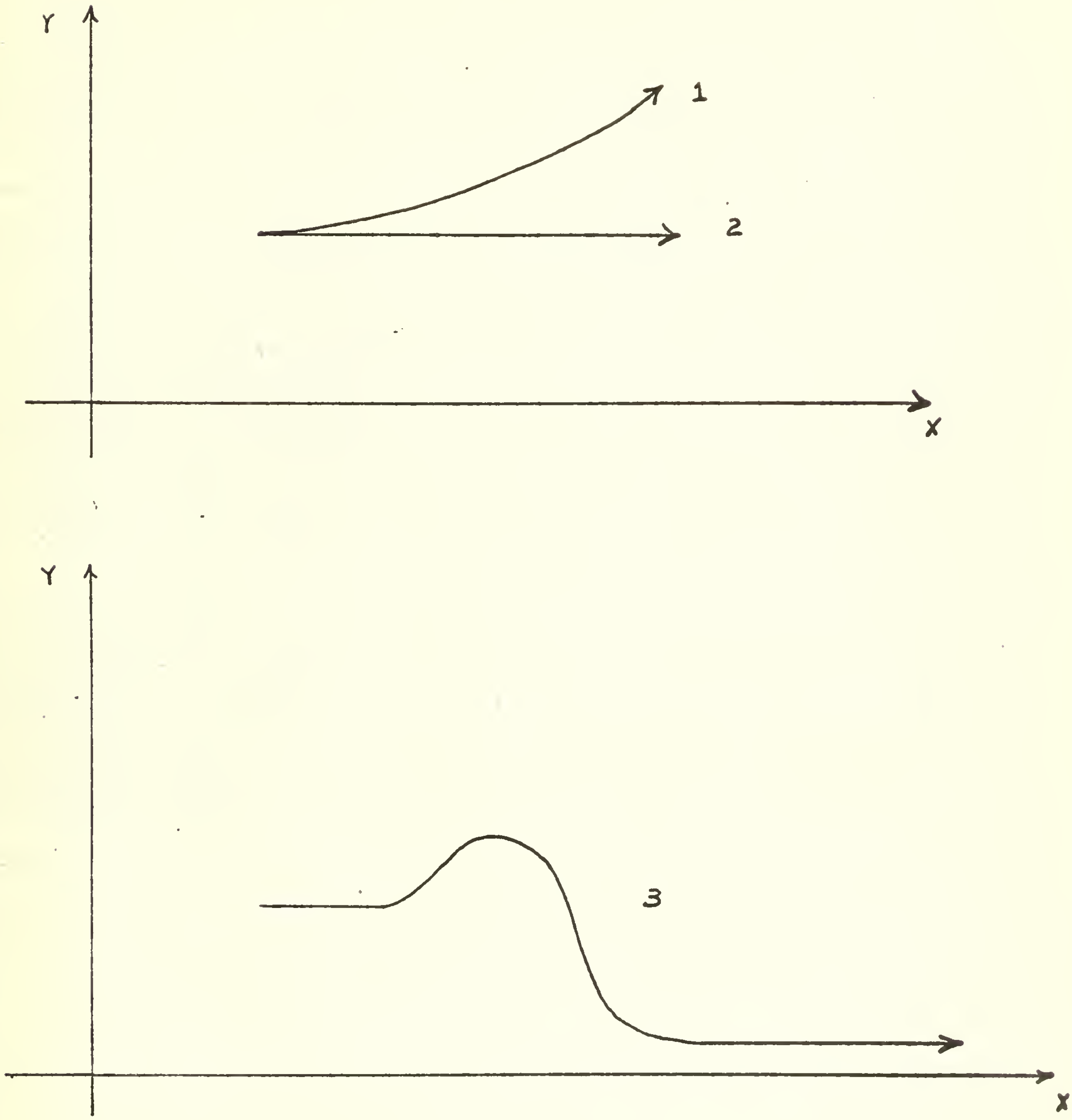


Figure 5-1 INITIAL CONDITION OF TARGET

Target velocity direction: A,B,C,D

Missile: Same initial position for all runs



EVASION COURSES FOR TARGET

Figure 5-2 TARGET TRAJECTORIES

Table 5-1

INITIAL CONDITIONS

	A-1	B-1	C-1	D-1	
V_m	3000 *	3000	3000	3000	
X_m	0	0	0	0	
Y_m	38000 #	38000	38000	38000	
R	30000 #	30000	30000	30000	
X_t	30000 #	30000	28300	28300	
Y_t	38000 #	38000	28000	28000	
$\dot{X}_t$	1000 *	-1000	1000	-1000	
$\dot{Y}_t$	0 *	0	0	0	
σ	0	0	-19.5°	-19.5°	
γ	0	0	-13.1°	-25.9°	
$\dot{R}$	2000 *	4000	2040	3930	

Velocity (Ft/sec) - *

Position (Ft.) - #

and crosscoupling could be considered. The trajectories may be considered as yaw or pitch maneuvers. If they are considered pitch maneuvers, the effects of altitude variations and gravity must be considered. Otherwise it is assumed that the missile has the same dynamic reaction to a change in either plane. The gravity consideration was not considered in this simulation.

The target was made to under-go several maneuvers: (1) a turn, (2) a straight line course, and (3) a left/right/left turn (evasive course). Figure 5-1 is a graphic representation of the missile and target initial conditions where A, B, C, D represents the target velocity vector for the four cases. The numerical values of all the parameters involved are listed in Table 5-1. The different maneuvers of the target are shown in Figure 5-2. A particular computer run will be designated (A-1-2-0.1-0.1). The "A" implies the initial conditions of the target (position and velocity direction). The first "1" implies that the magnitude of the target's velocity is 1000 ft/sec. The "2" implies that the target is flying a straight line course (maneuver two). The last two numbers are the standard deviation of the excitation noise and the measurement noise respectively. Note that all angles in this program are given in radians.

Figure 5-3 to Figure 5-26 are presented below to display the filtered missile capabilities. Listed on each figure are the range at which the program was terminated ($R < 100$ ft.) and the predicted miss distance.

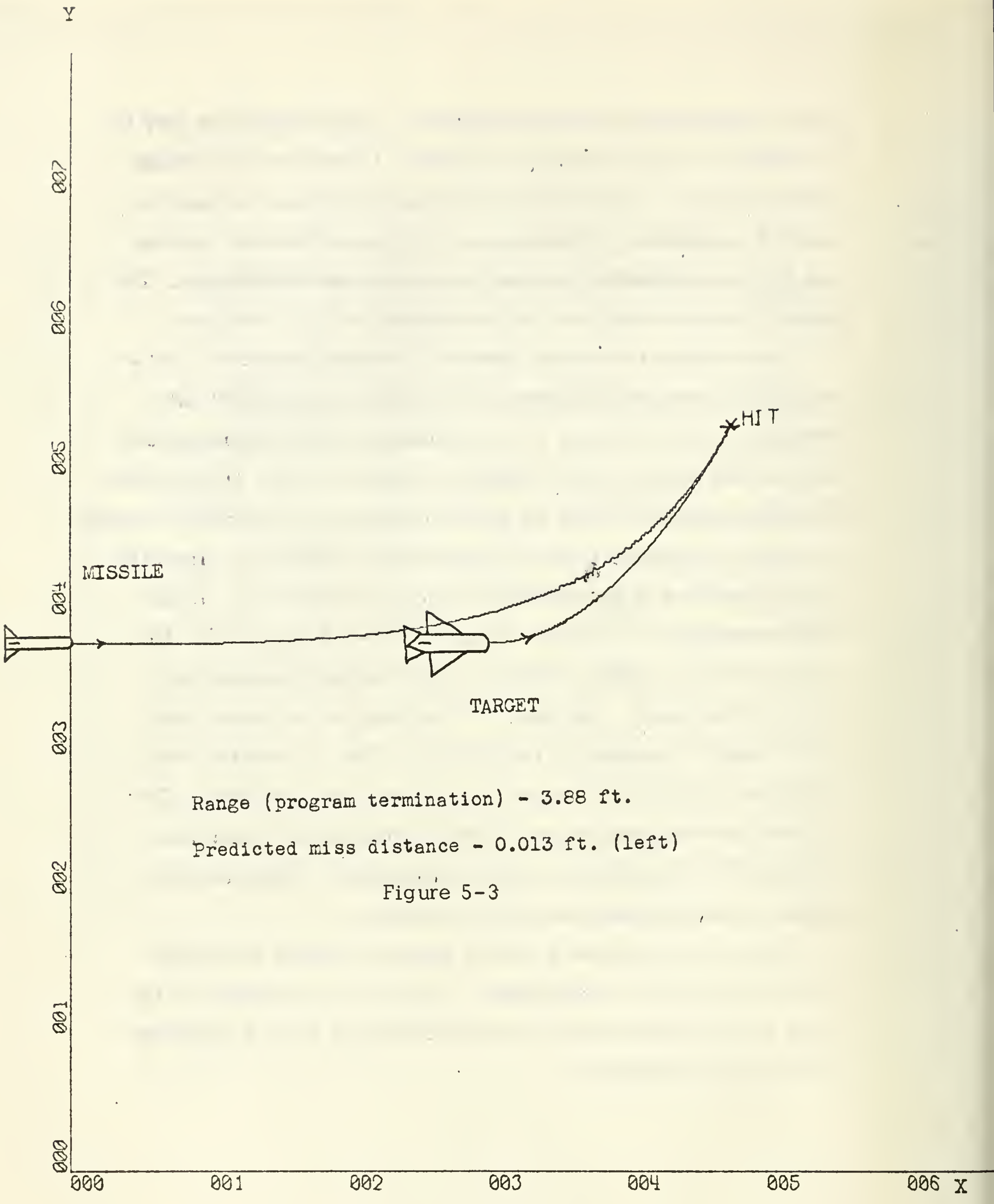


Figure 5-3

X-SCALE = 1.00E+04 UNITS/INCH. (units - feet)
Y-SCALE = 1.00E+04 UNITS/INCH.

MISSILE ACTIVE GUIDANCE SIMULATION
A-1-1-0.01-0.01

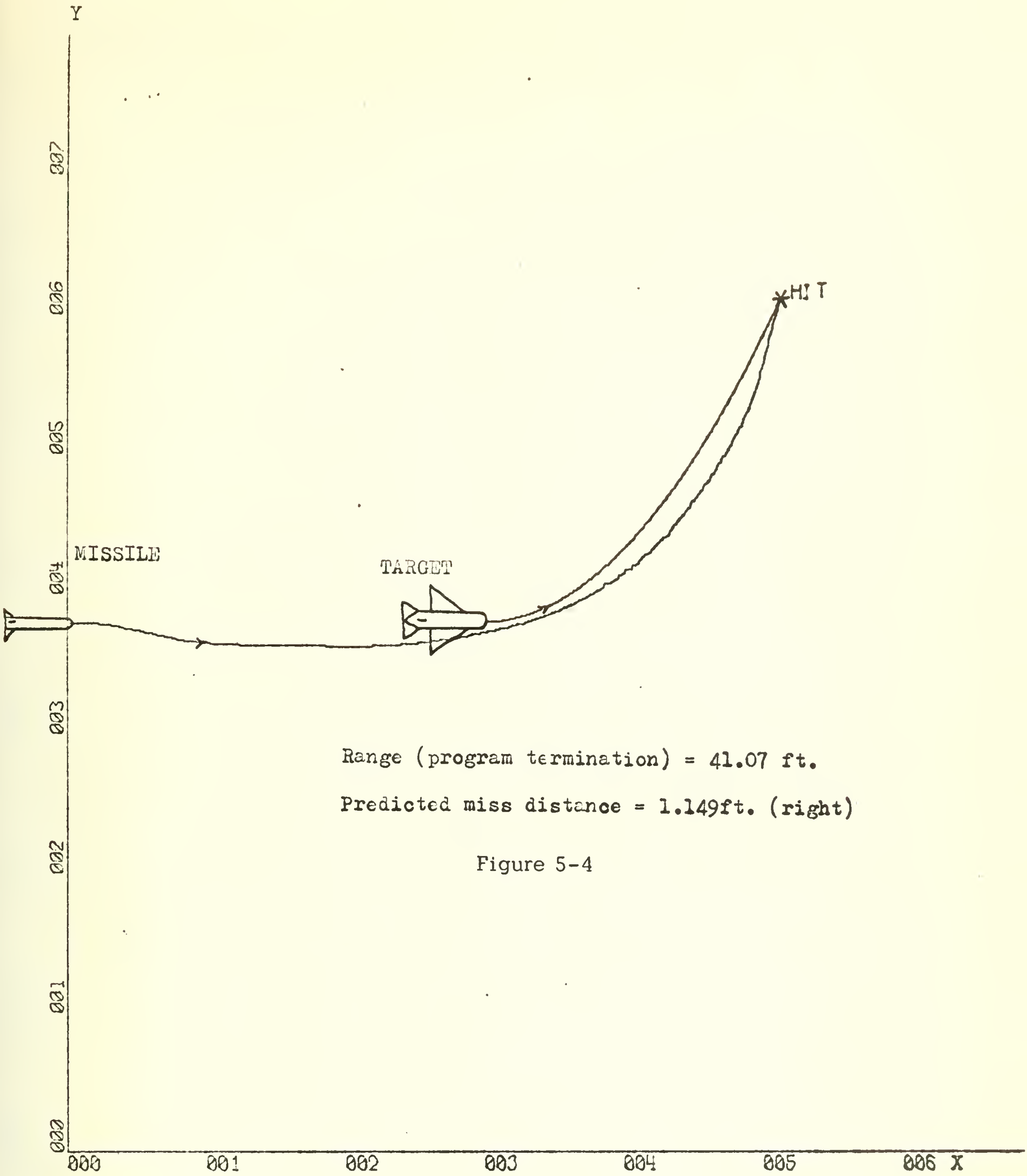


Figure 5-4

X-SCALE = 1.00E+04 UNITS/INCH. (units = feet)
Y-SCALE = 1.00E+04 UNITS/INCH.

SIMULATION OF GUIDED MISSILE/FILTER(STEELE)
A-1-1-0.1-0.1

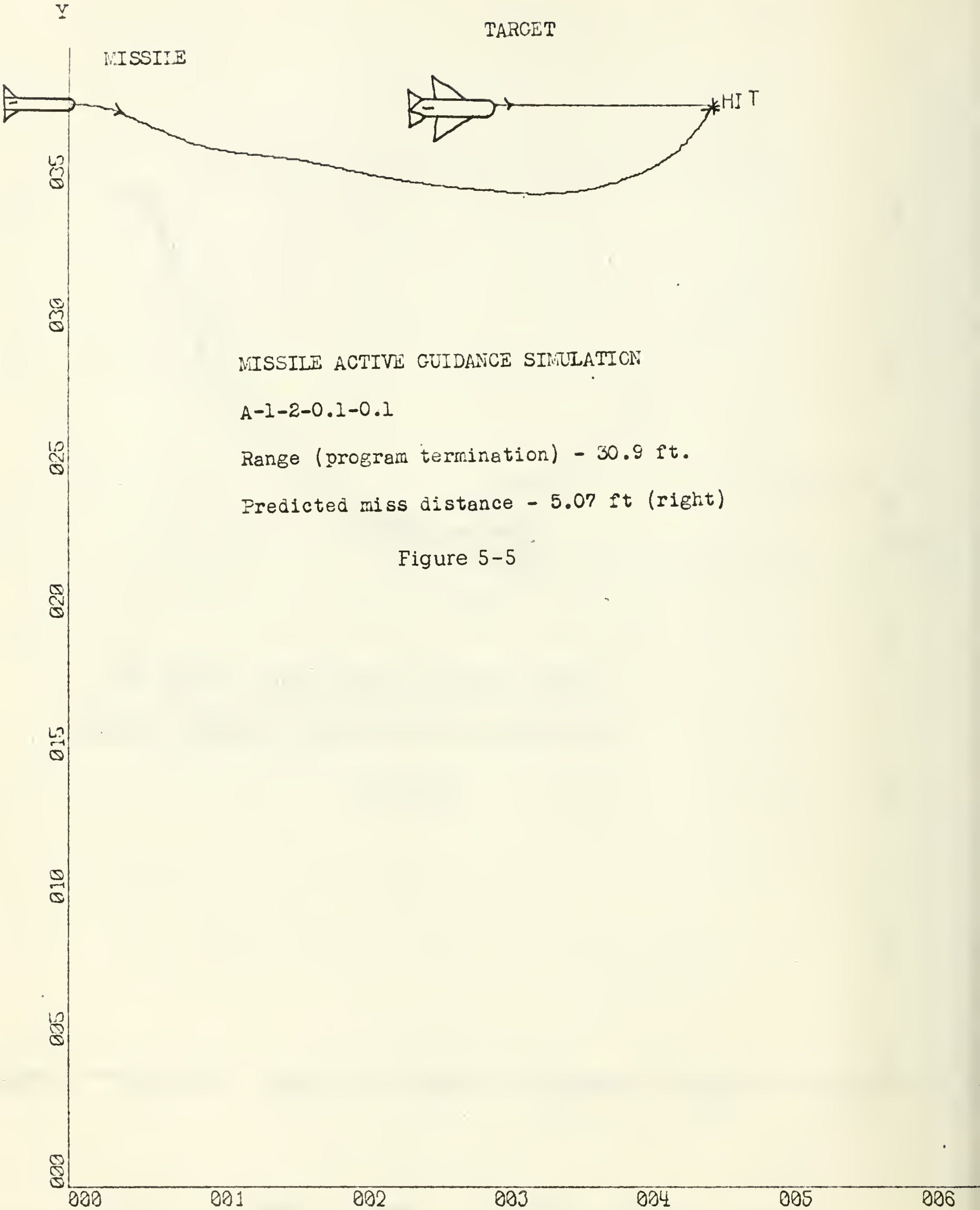


Figure 5-5

X-SCALE = 1.00E+04 UNITS/INCH. (units - feet)
Y-SCALE = 5.00E+03 UNITS/INCH.

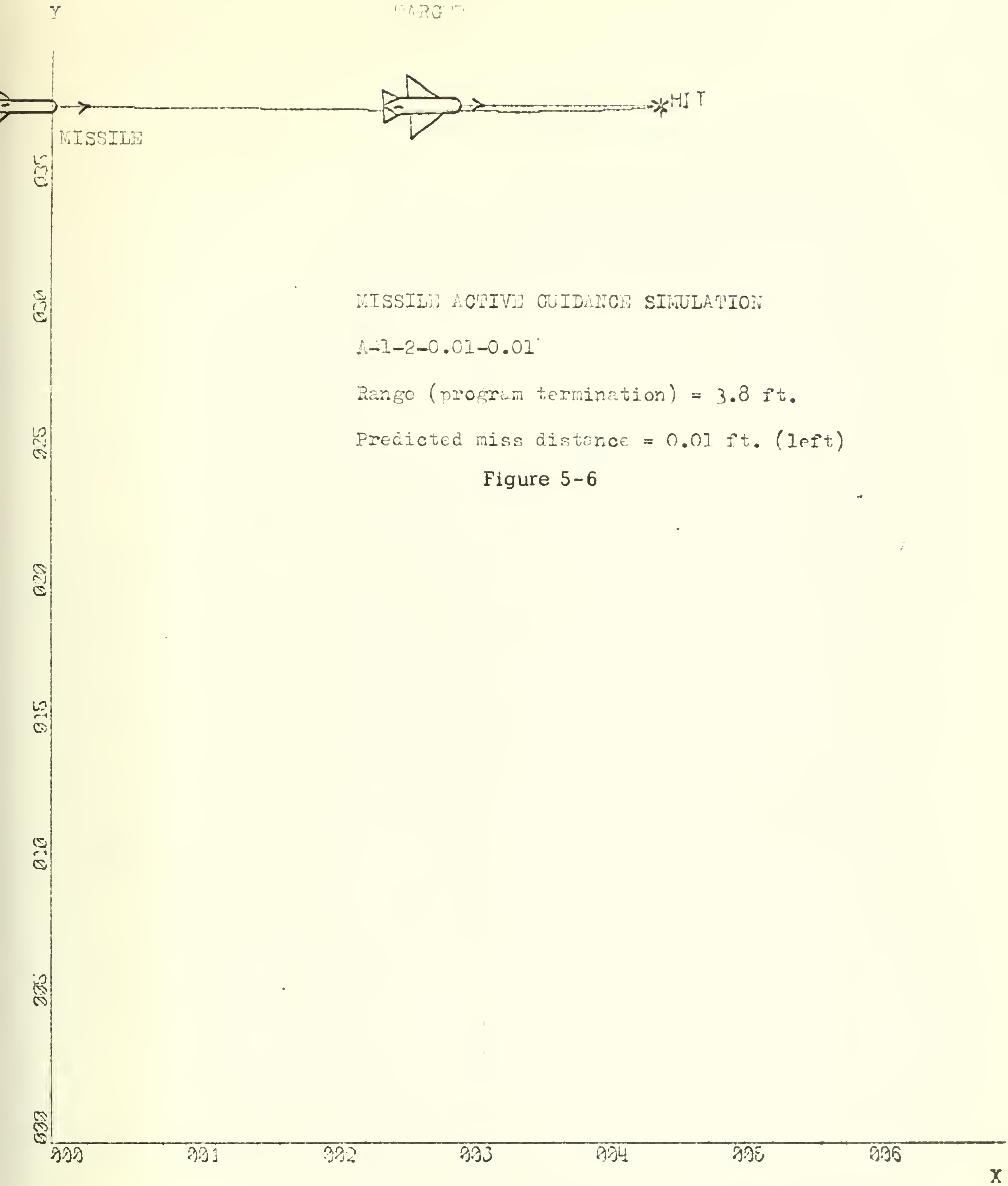


Figure 5-6

X-SCALE = 1.00E+04 UNITS/INCH. (Units = feet)
Y-SCALE = 5.00E+03 UNITS/INCH.

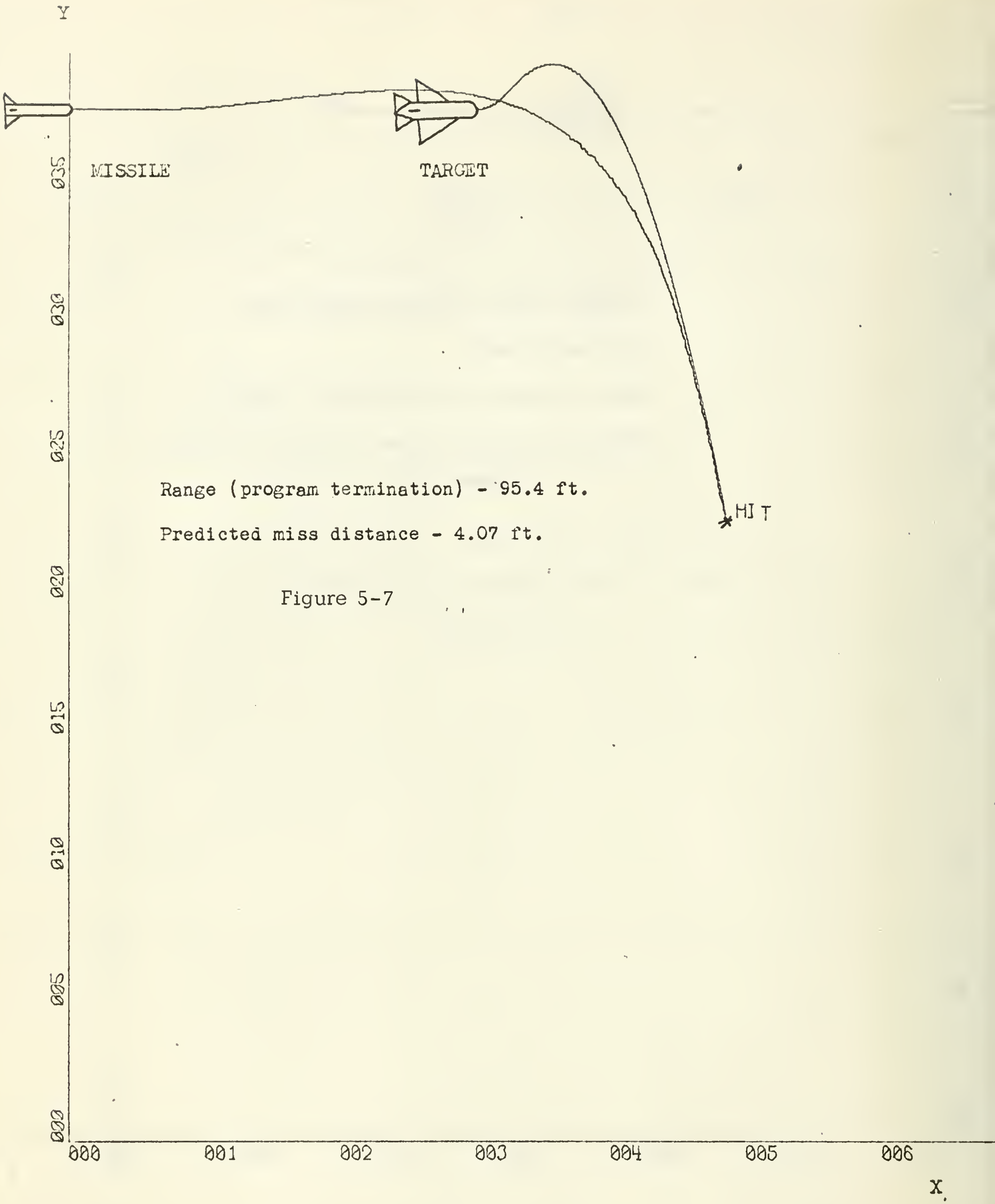
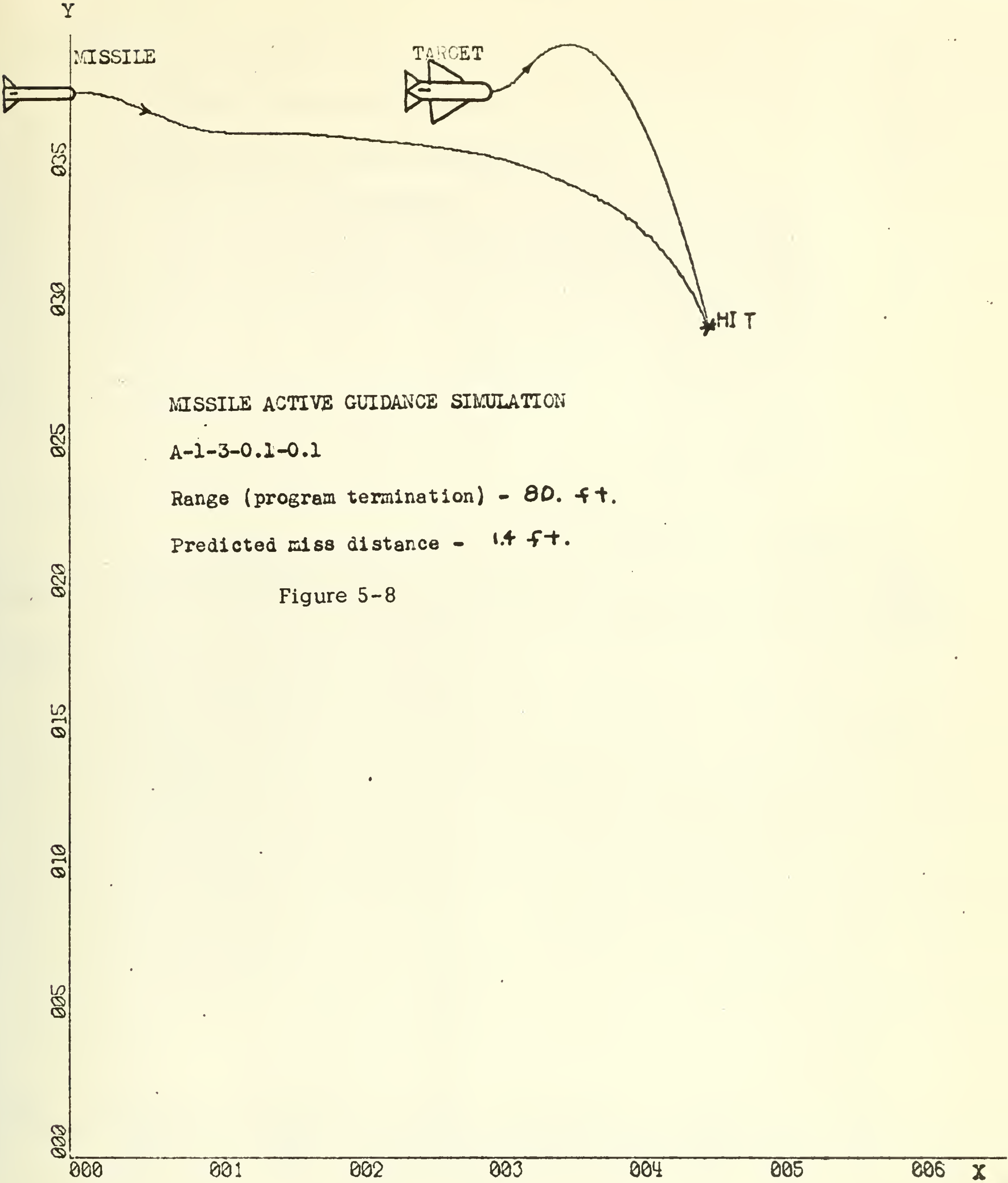


Figure 5-7

X-SCALE = 1.00E+04 UNITS/INCH. (units - feet)
Y-SCALE = 5.00E+03 UNITS/INCH.

SIMULATION OF GUIDED MISSILE/FILTER(STEELE)
A-1-3-0.01-0.01 STEELE



MISSILE ACTIVE GUIDANCE SIMULATION

A-1-3-0.1-0.1

Range (program termination) - 80. ft.

Predicted miss distance - 1.4 ft.

Figure 5-8

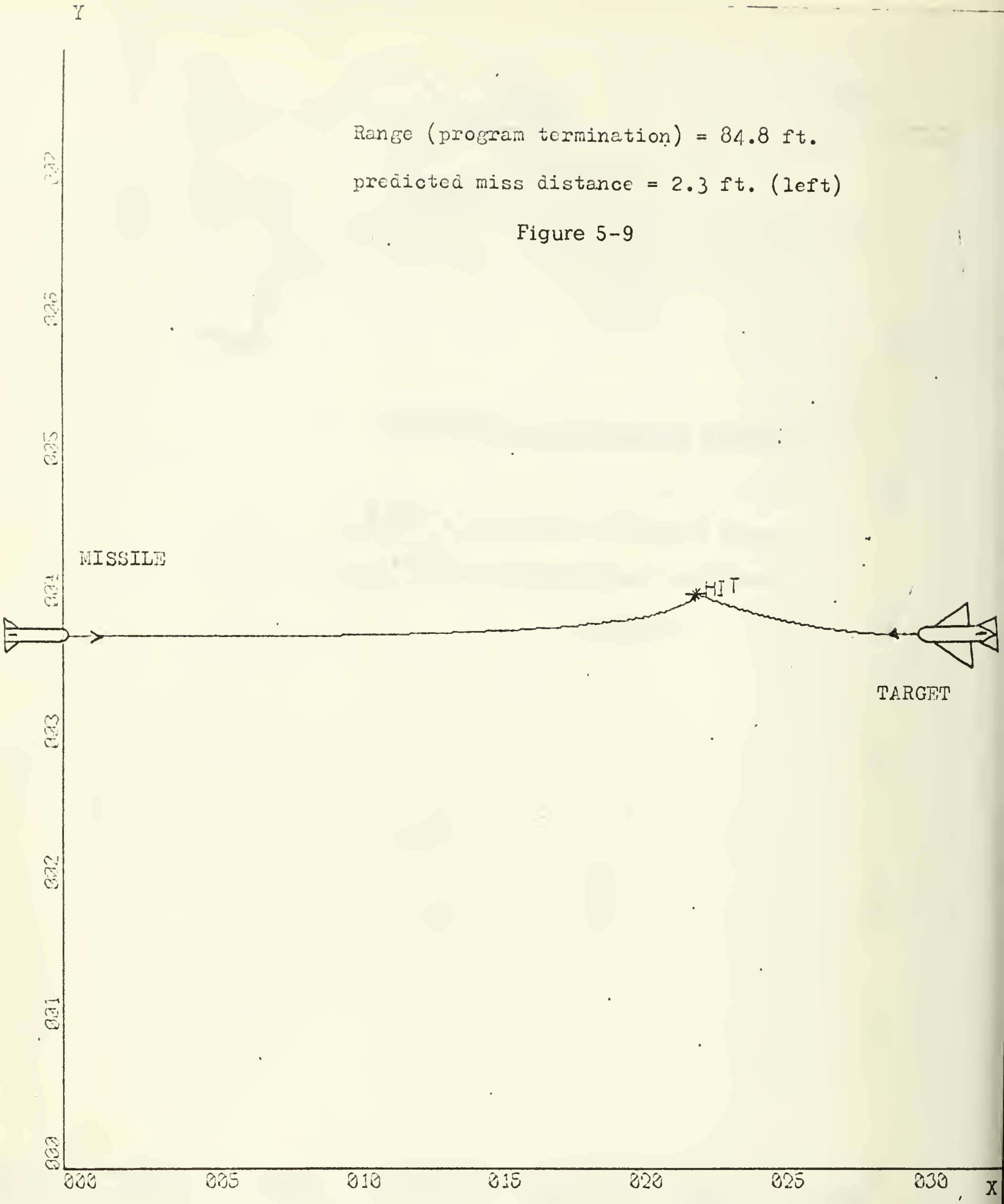
X-SCALE = 1.00E+04 UNITS/INCH. (units - feet)

Y-SCALE = 5.00E+03 UNITS/INCH.

Range (program termination) = 84.8 ft.

predicted miss distance = 2.3 ft. (left)

Figure 5-9



X-SCALE = 5.00E+03 UNITS/INCH. (Units = feet)

Y-SCALE = 1.00E+04 UNITS/INCH.

MISSILE ACTIVE GUIDANCE SIMULATION

B-1-1-0.01-0.01

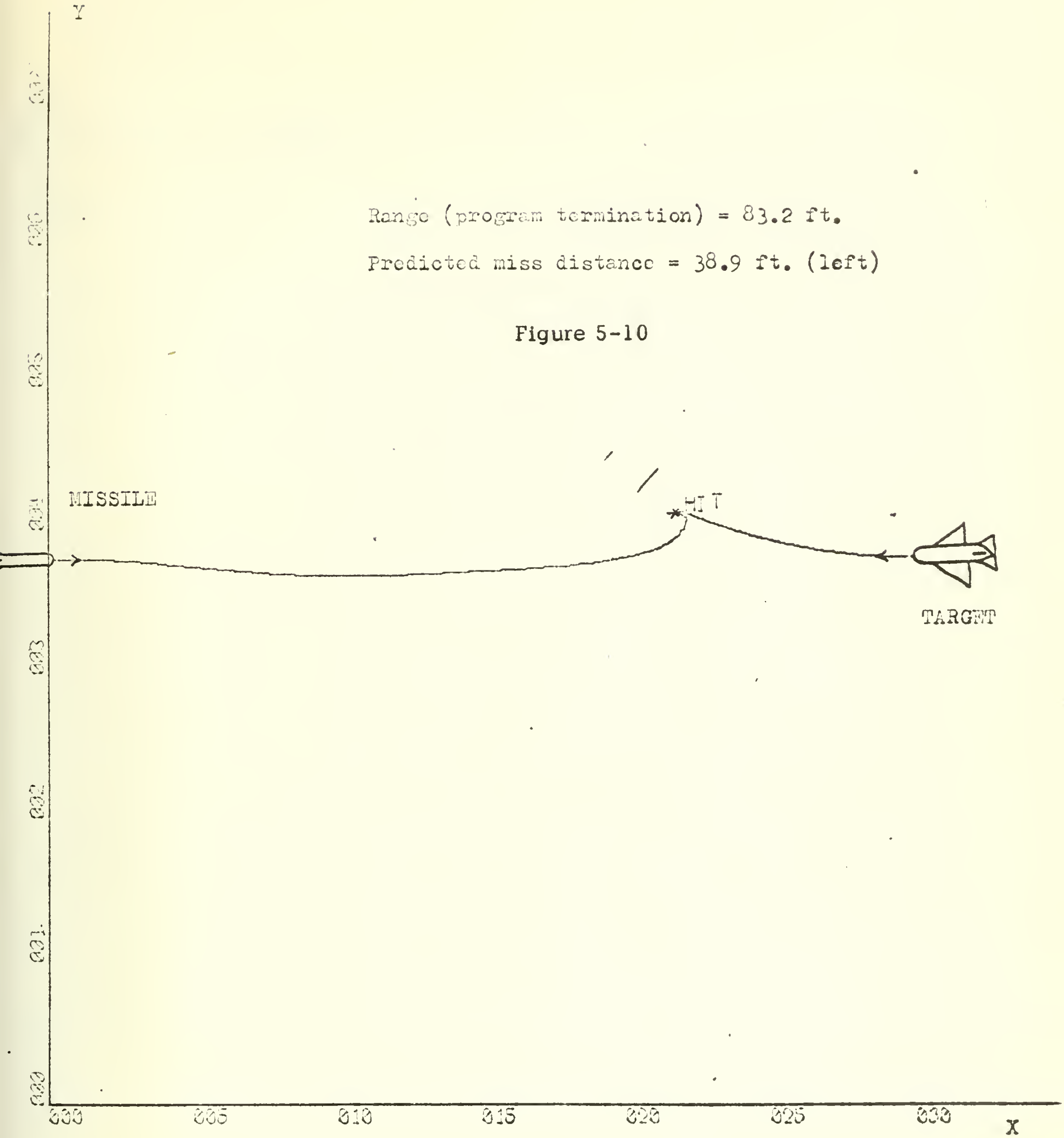
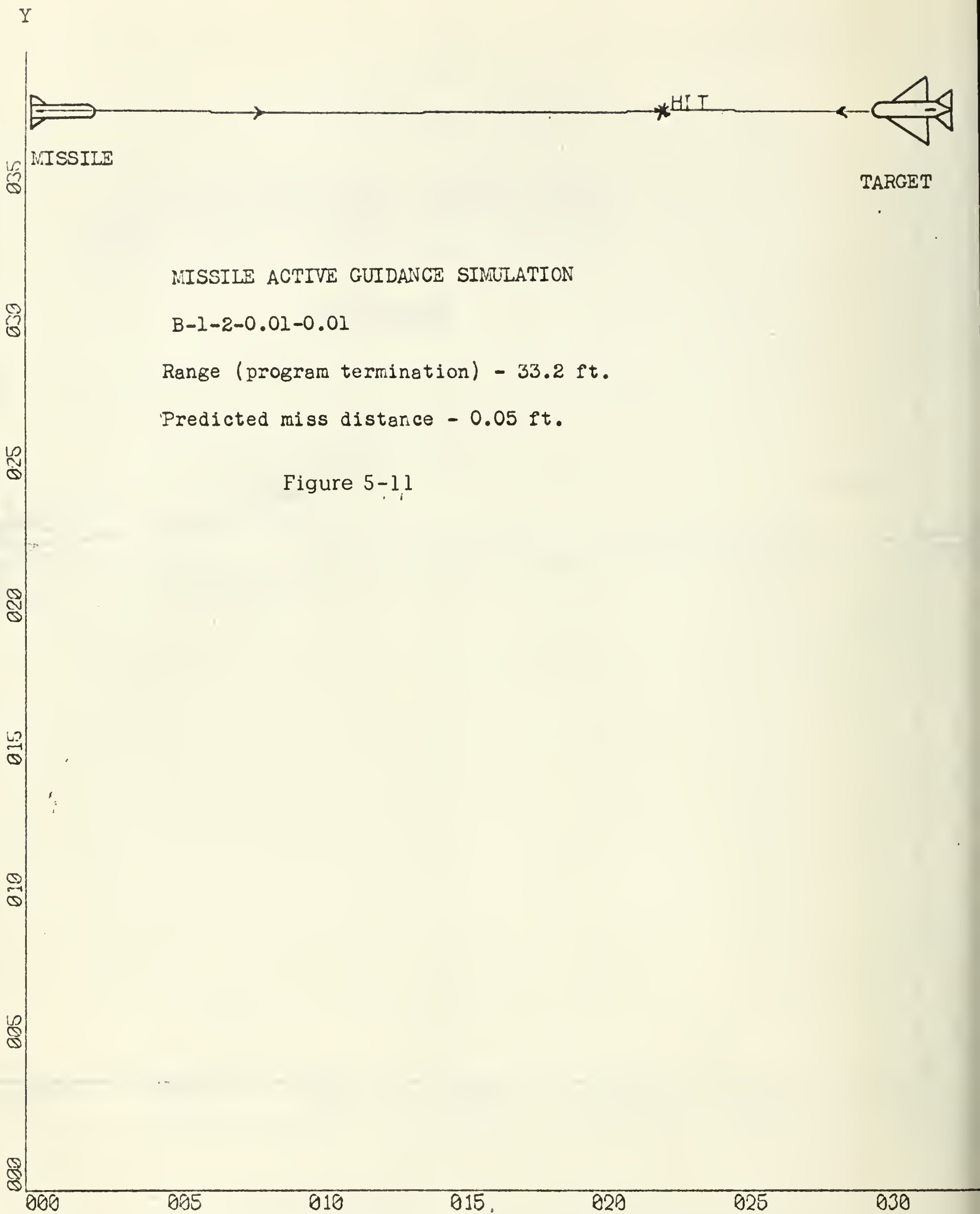


Figure 5-10

(Units = feet)

X-SCALE = 5.00E+03 UNITS/INCH.
Y-SCALE = 1.00E+04 UNITS/INCH.

MISSILE ACTIVE GUIDANCE SIMULATION
B-1-1-0.1-0.1



MISSILE ACTIVE GUIDANCE SIMULATION

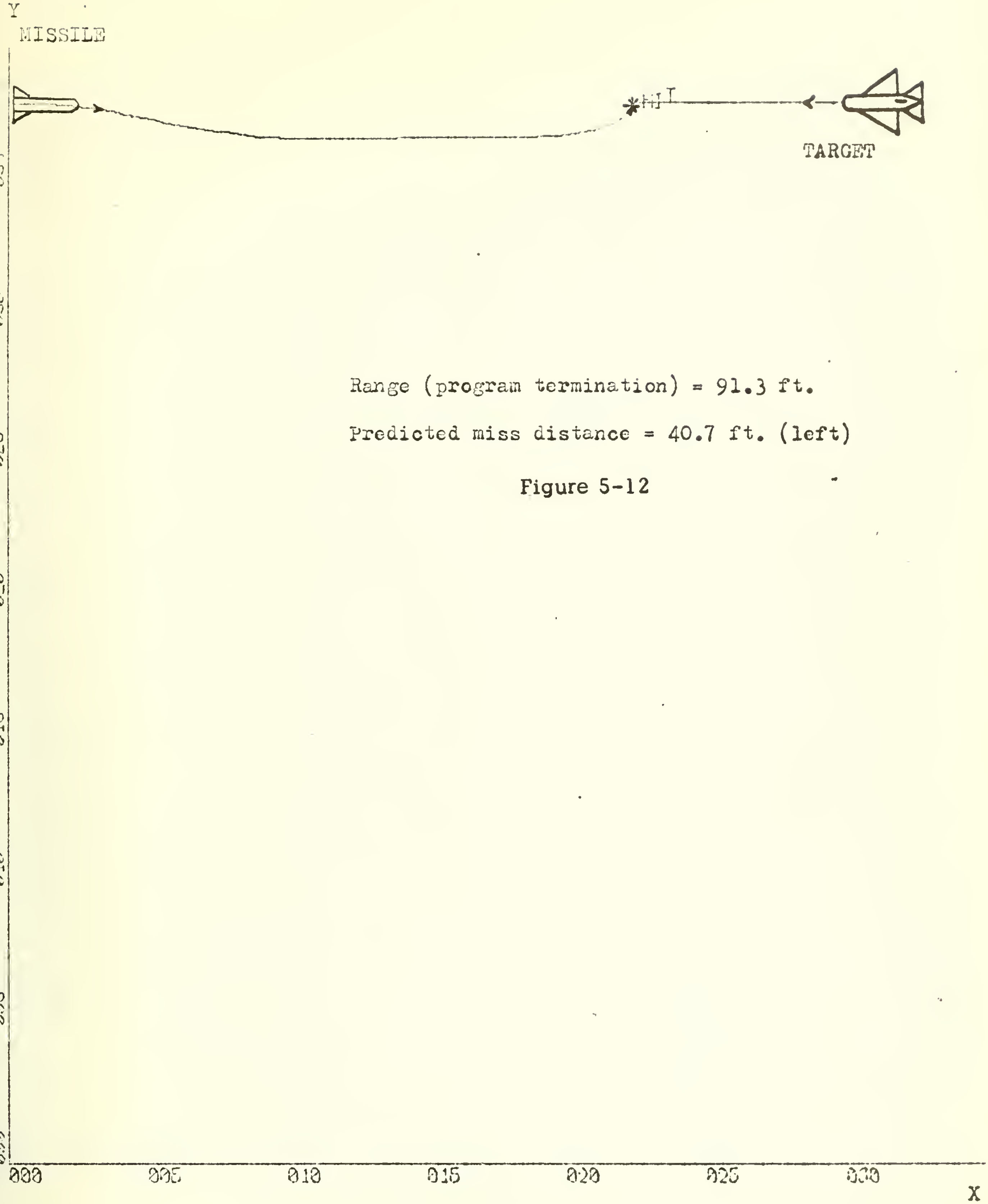
B-1-2-0.01-0.01

Range (program termination) - 33.2 ft.

Predicted miss distance - 0.05 ft.

Figure 5-11

X-SCALE = 5.00E+03 UNITS/INCH.
Y-SCALE = 5.00E+03 UNITS/INCH.



Range (program termination) = 91.3 ft.

Predicted miss distance = 40.7 ft. (left)

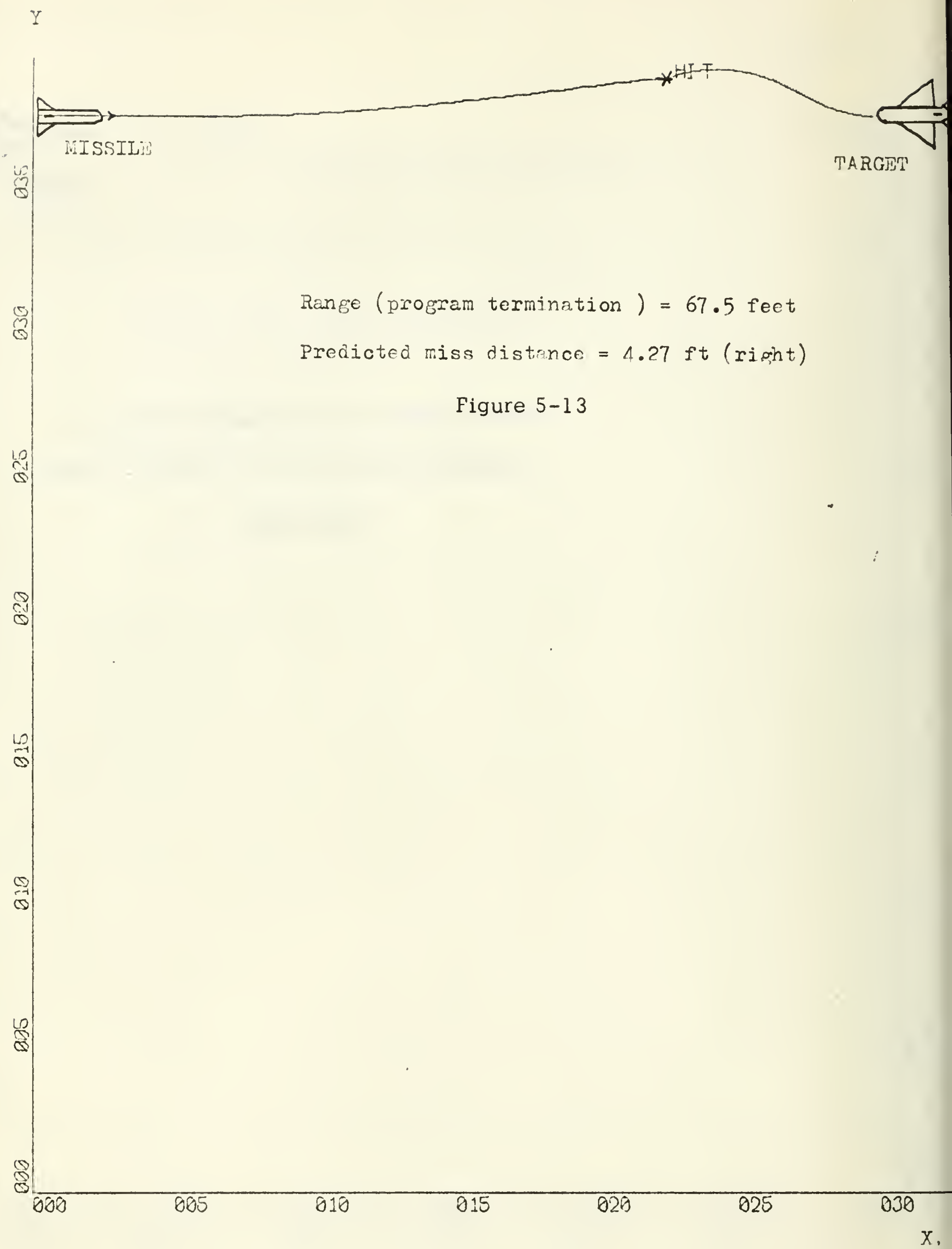
Figure 5-12

(units = feet)

X-SCALE = 5.00E+03 UNITS/INCH.
Y-SCALE = 5.00E+03 UNITS/INCH.

MISSILE ACTIVE GUIDANCE SIMULATION

R=1-2-0.1-0.1



Range (program termination) = 67.5 feet

Predicted miss distance = 4.27 ft (right)

Figure 5-13

X-SCALE = 5.00E+03 UNITS/INCH. (Units = feet)
Y-SCALE = 5.00E+03 UNITS/INCH.

MISSILE ACTIVE GUIDANCE SIMULATION
B-1-3-0.01-0.01

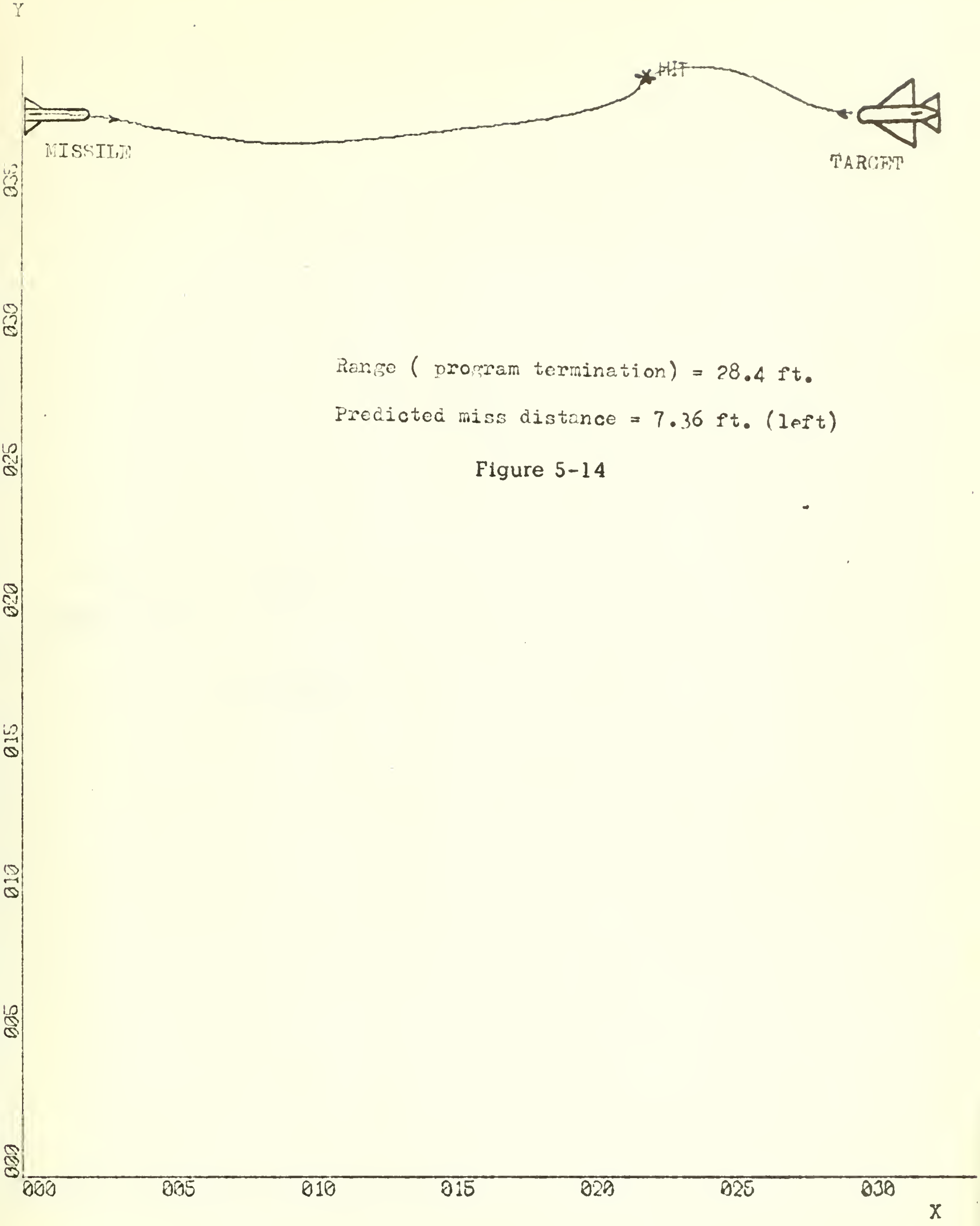


Figure 5-14

X-SCALE = 5.00E+03 UNITS/INCH. (Units = feet)
Y-SCALE = 5.00E+03 UNITS/INCH.

MISSILE ACTIVE GUIDANCE SIMULATION

B-1-3-0.1-0.1

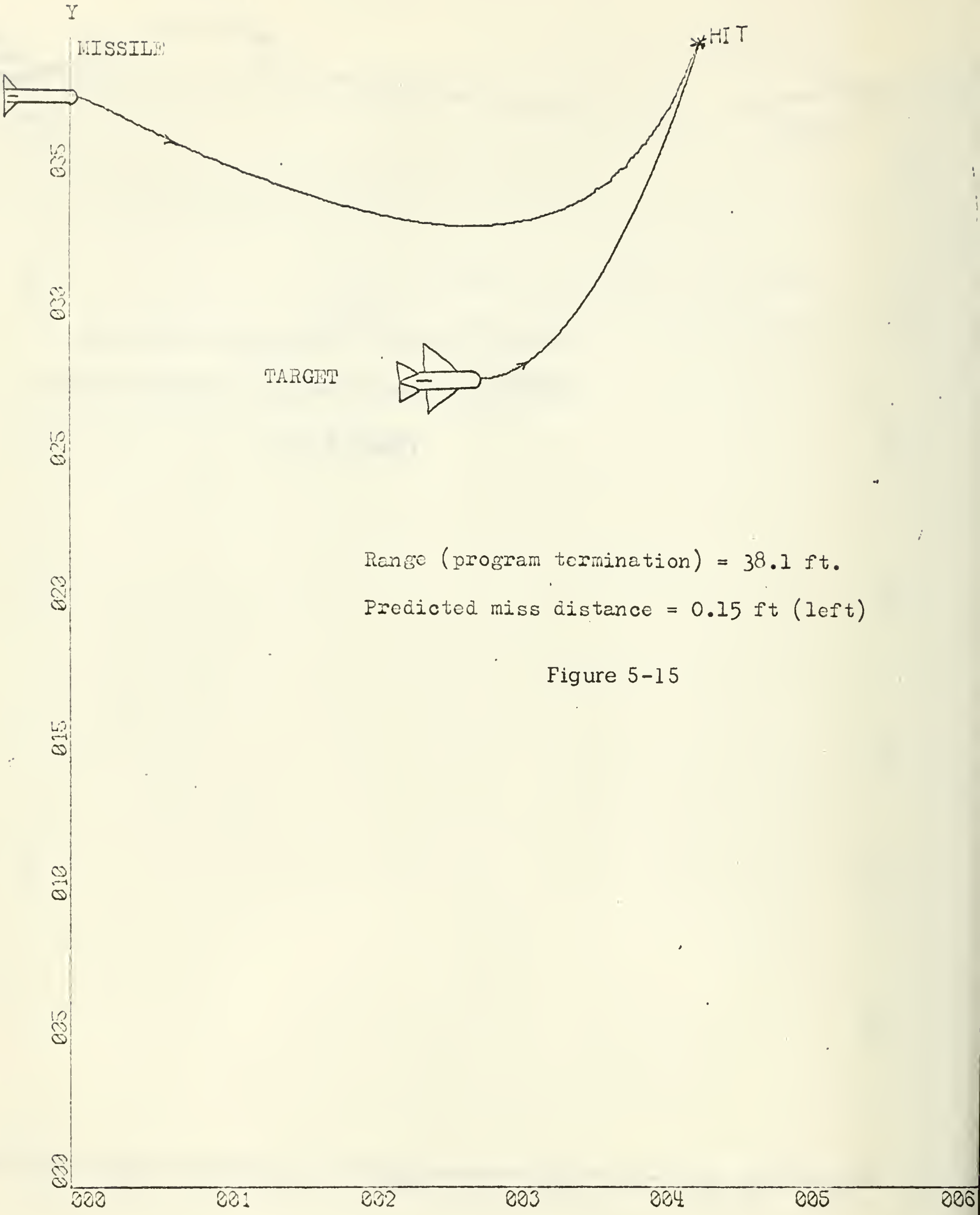


Figure 5-15

X-SCALE = 1.00E+04 UNITS/INCH. (Units = feet)
Y-SCALE = 5.00E+03 UNITS/INCH.

MISSILE ACTIVE GUIDANCE SIMULATION

C-1-1-0.01-0.01

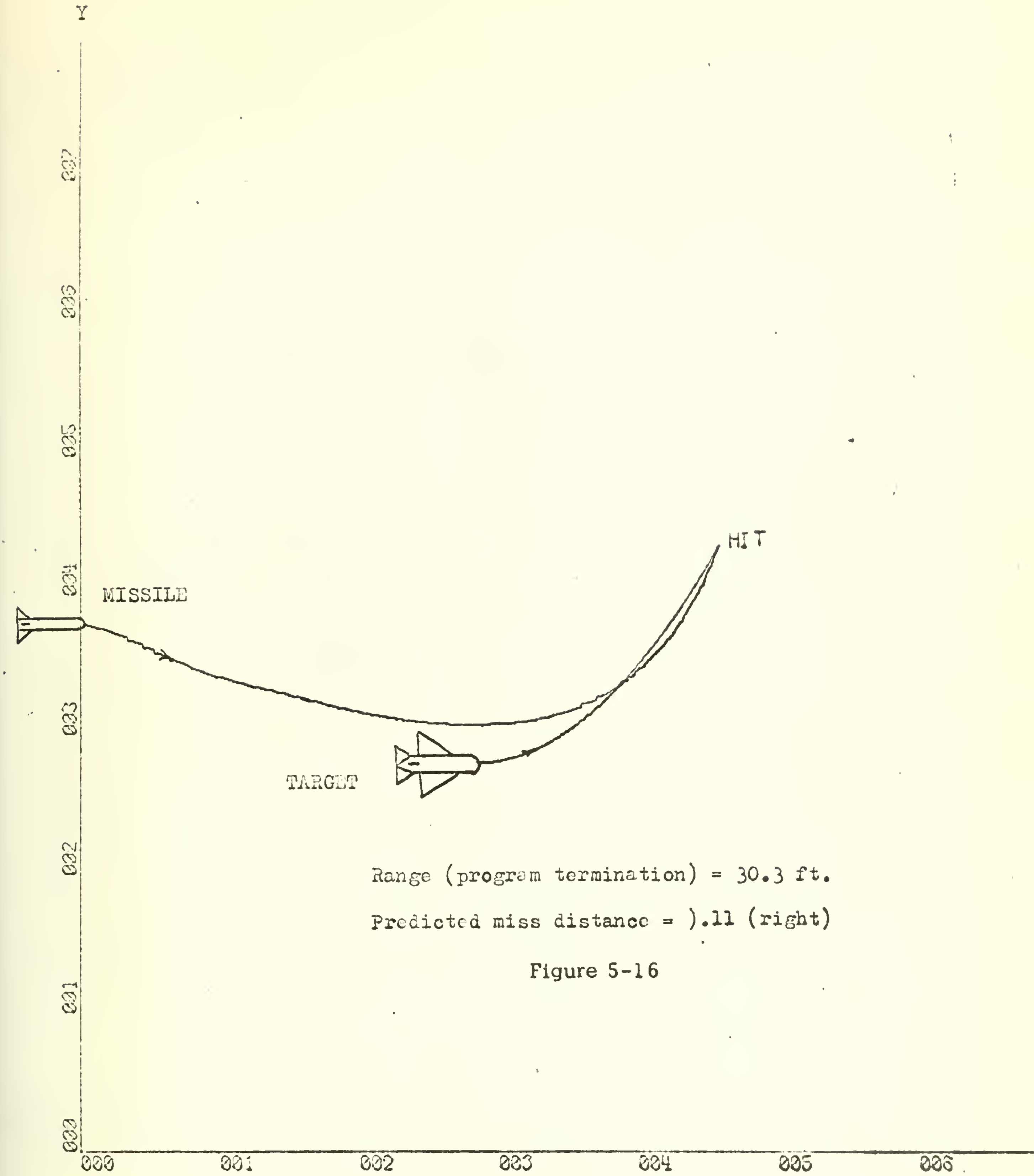


Figure 5-16

X-SCALE = 1.00E+04 UNITS/INCH.
Y-SCALE = 1.00E+04 UNITS/INCH.

MISSILE ACTIVE GUIDANCE SIMULATION

C-1-1-0.1-0.1

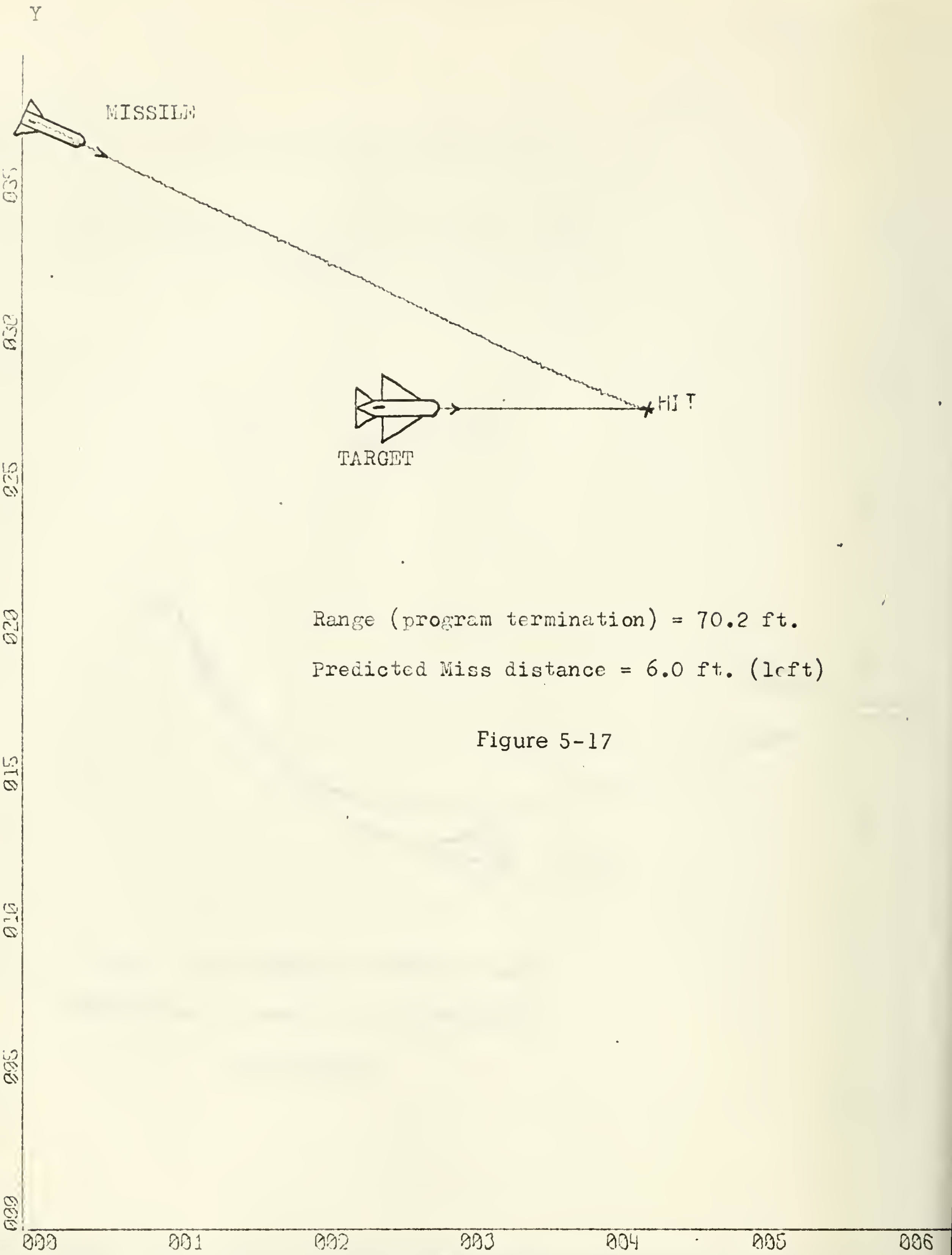


Figure 5-17

X-SCALE = 1.00E+04 UNITS/INCH.
Y-SCALE = 5.00E+03 UNITS/INCH.

(Units = feet)

MISSILE ACTIVE GUIDANCE SIMULATION
C-1-2-0.01-0.01

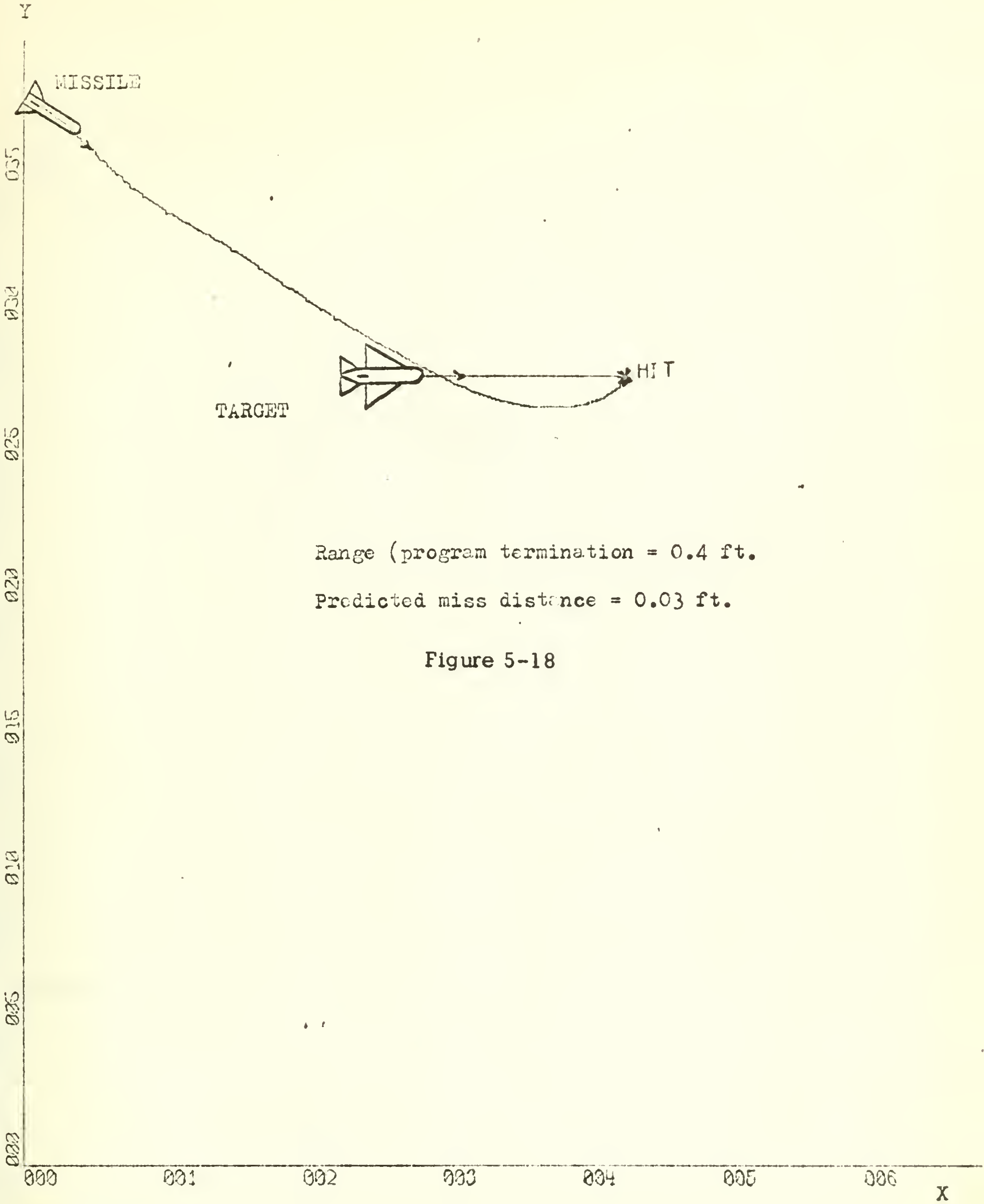


Figure 5-18

(units = feet)

X-SCALE = 1.00E+04 UNITS/INCH.
Y-SCALE = 5.00E+03 UNITS/INCH.

MISSILE ACTIVE GUIDANCE SIMULATION
C-1-2-0.1-0.1

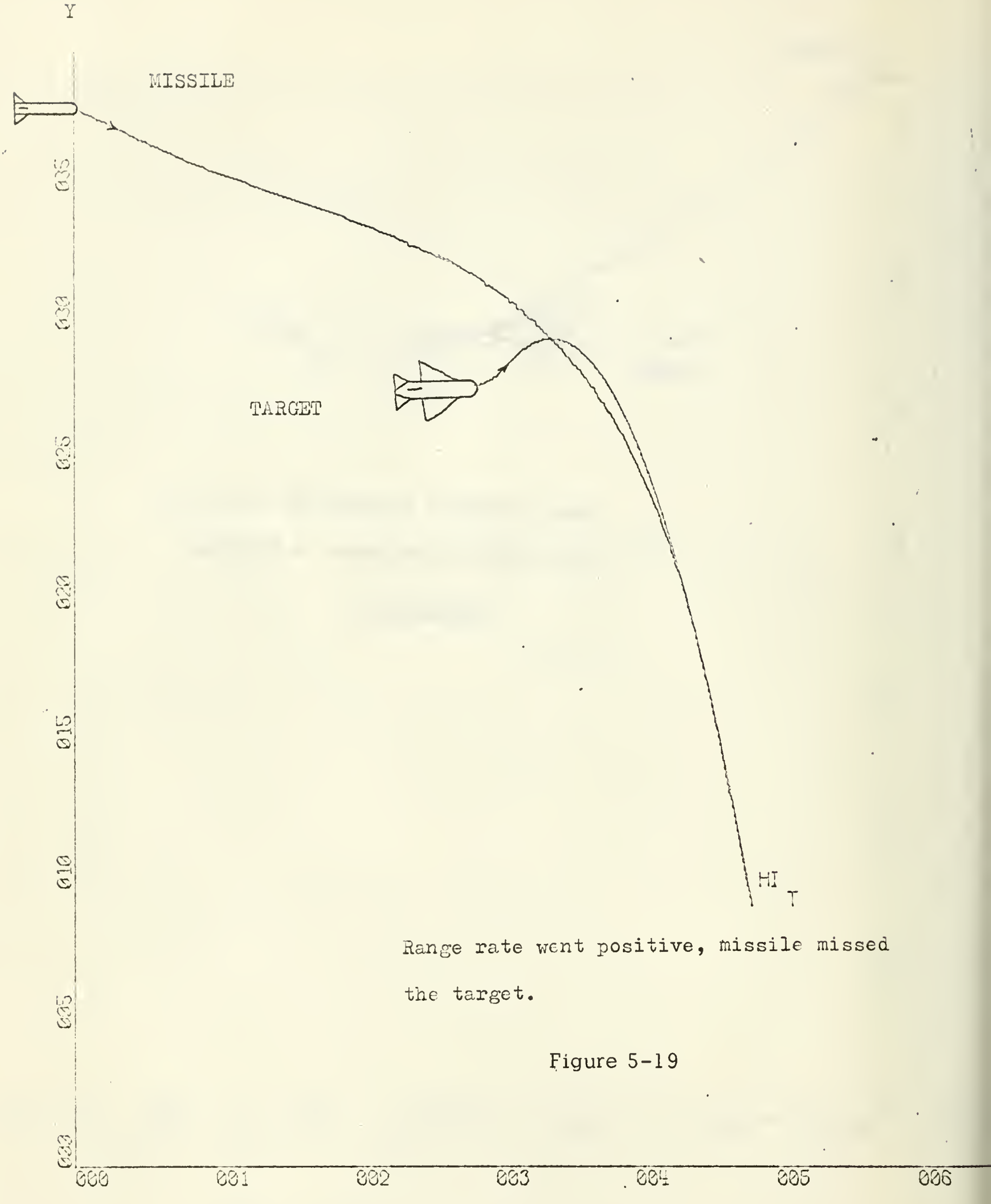


Figure 5-19

(units = feet)

X-SCALE = 1.00E+04 UNITS/INCH.
Y-SCALE = 5.00E+03 UNITS/INCH.

MISSILE ACTIVE GUIDANCE SIMULATION

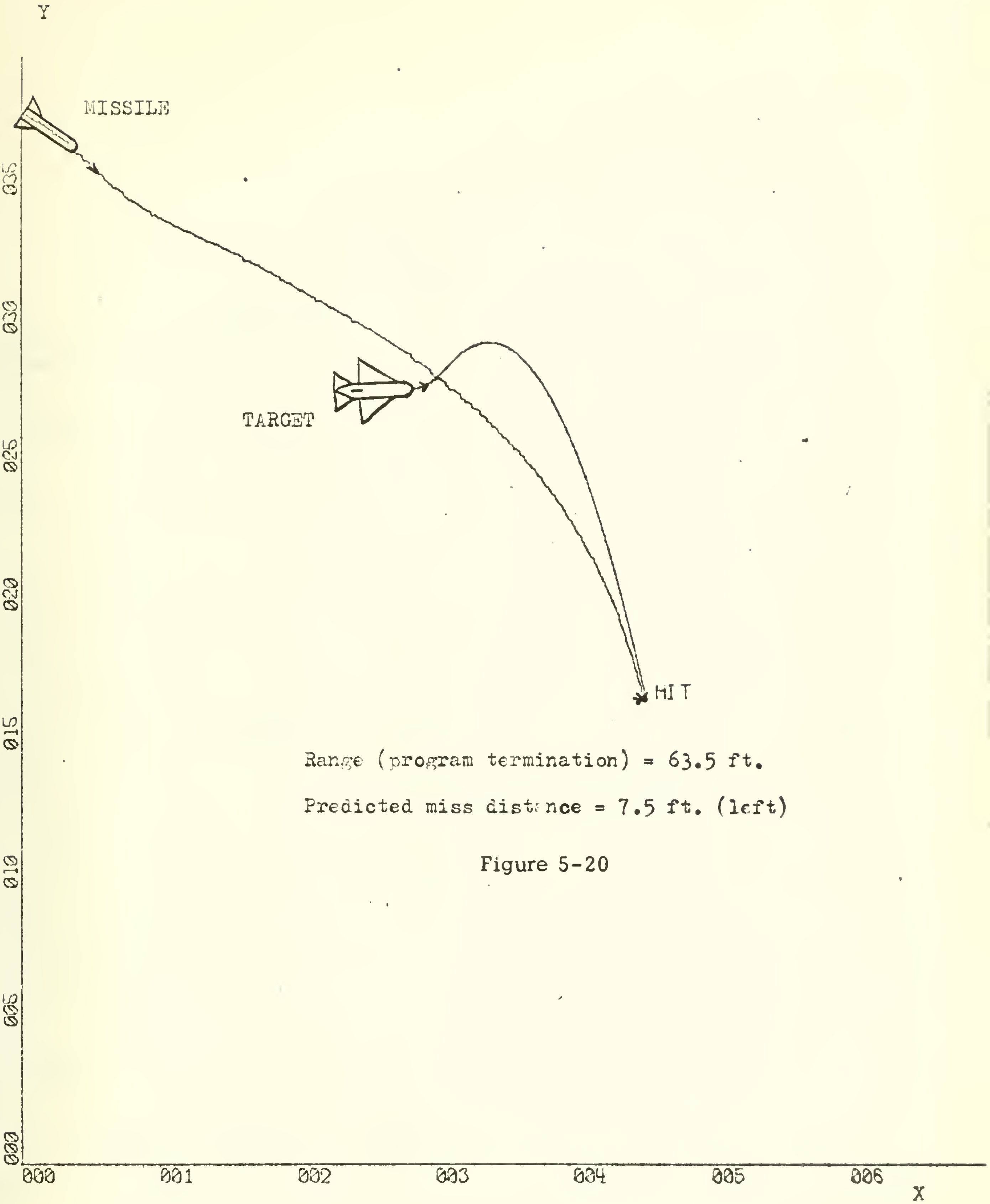
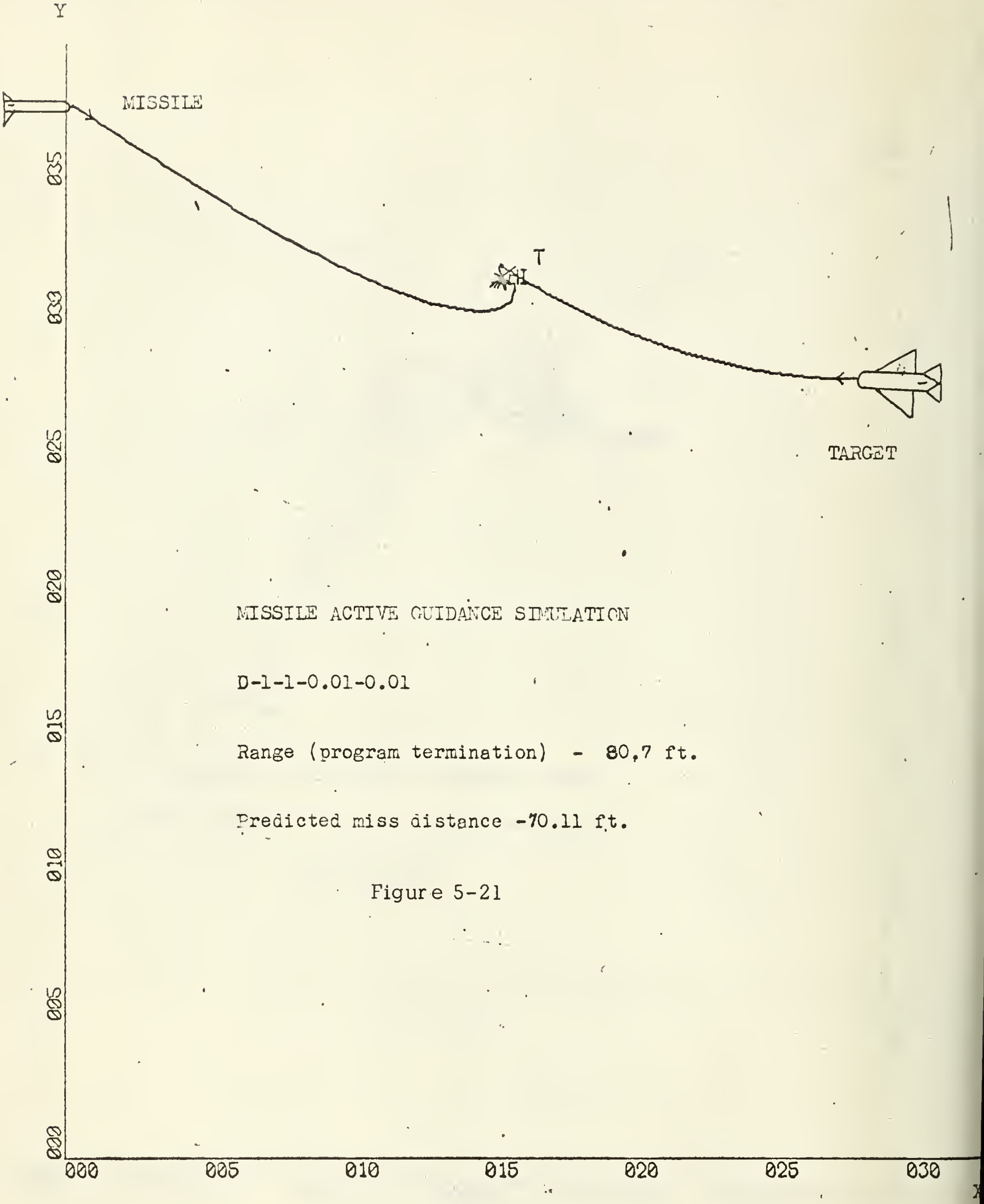


Figure 5-20

(units = feet)

X-SCALE = 1.00E+04 UNITS/INCH.
Y-SCALE = 5.00E+03 UNITS/INCH.

MISSILE ACTIVE GUIDANCE SIMULATION



MISSILE ACTIVE GUIDANCE SIMULATION

D-1-1-0.01-0.01

Range (program termination) - 80,7 ft.

Predicted miss distance -70.11 ft.

Figure 5-21

X-SCALE = 5.00E+03 UNITS/INCH.
Y-SCALE = 5.00E+03 UNITS/INCH. (units - feet)

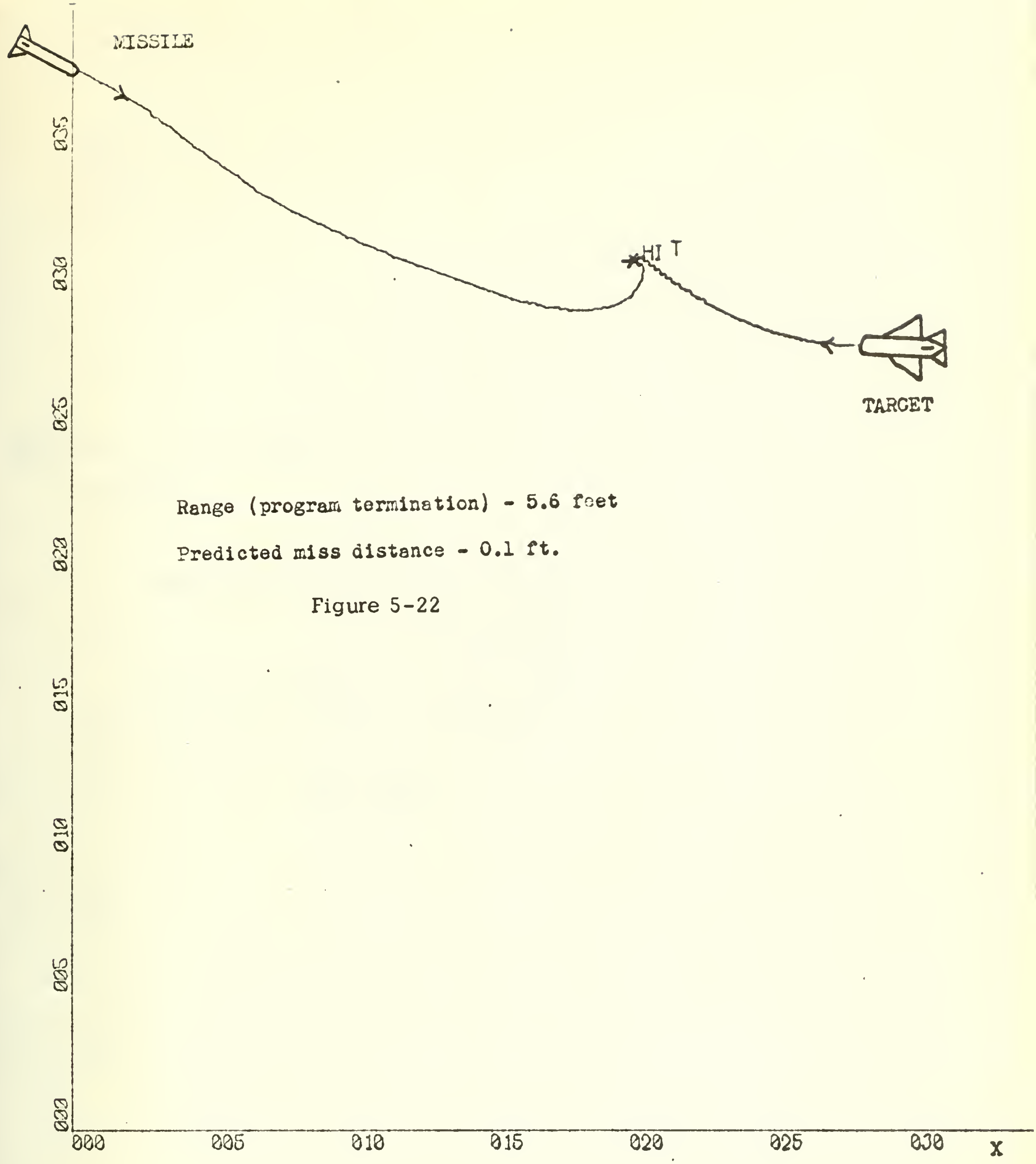


Figure 5-22

X-SCALE = 5.00E+03 UNITS/INCH. (units - feet)
Y-SCALE = 5.00E+03 UNITS/INCH.

MISSILE ACTIVE GUIDANCE SIMULATION

D-1-1-0.1-0.1 STEELE

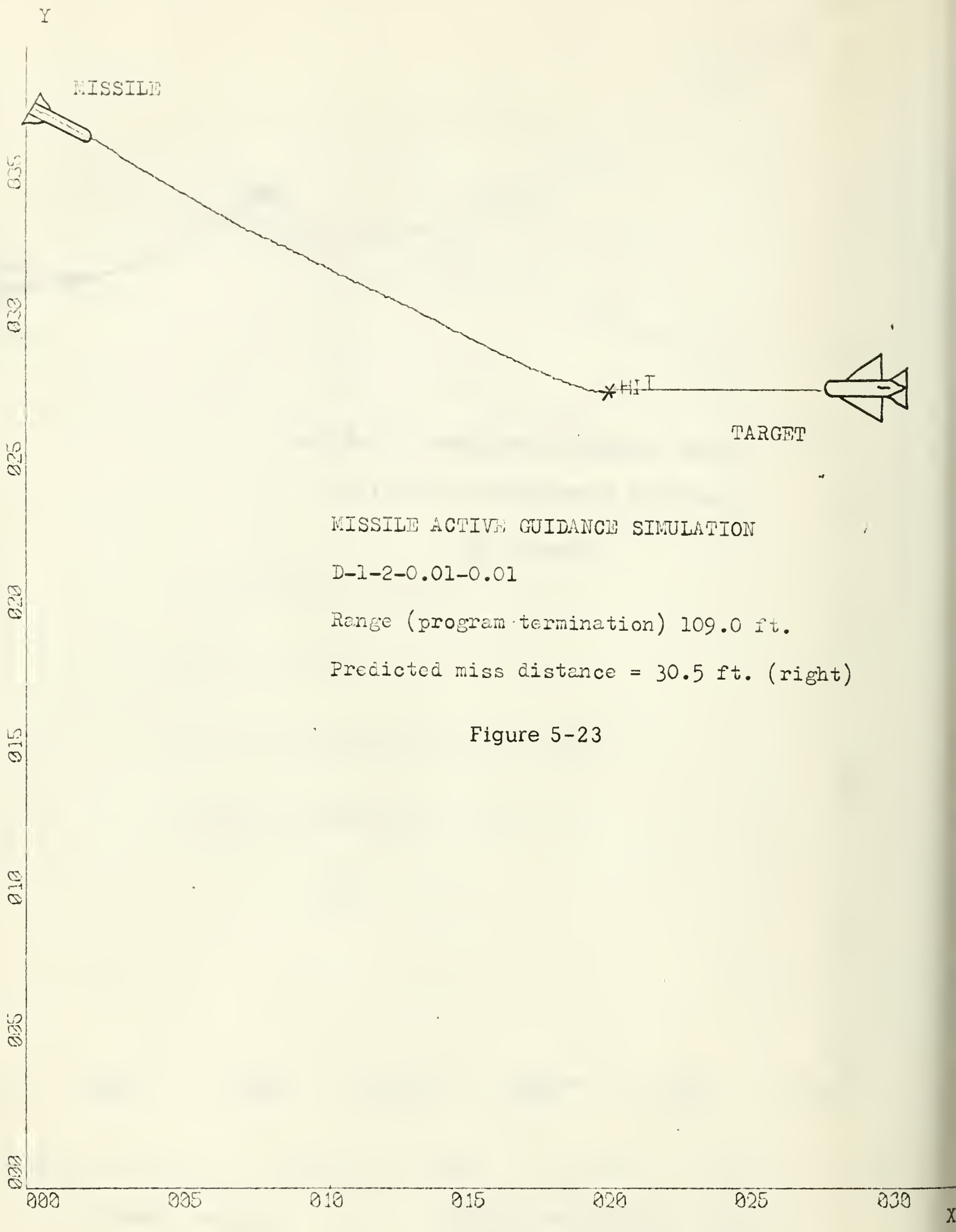


Figure 5-23

X-SCALE = 5.00E+03 UNITS/INCH. (units = feet)
Y-SCALE = 5.00E+03 UNITS/INCH.

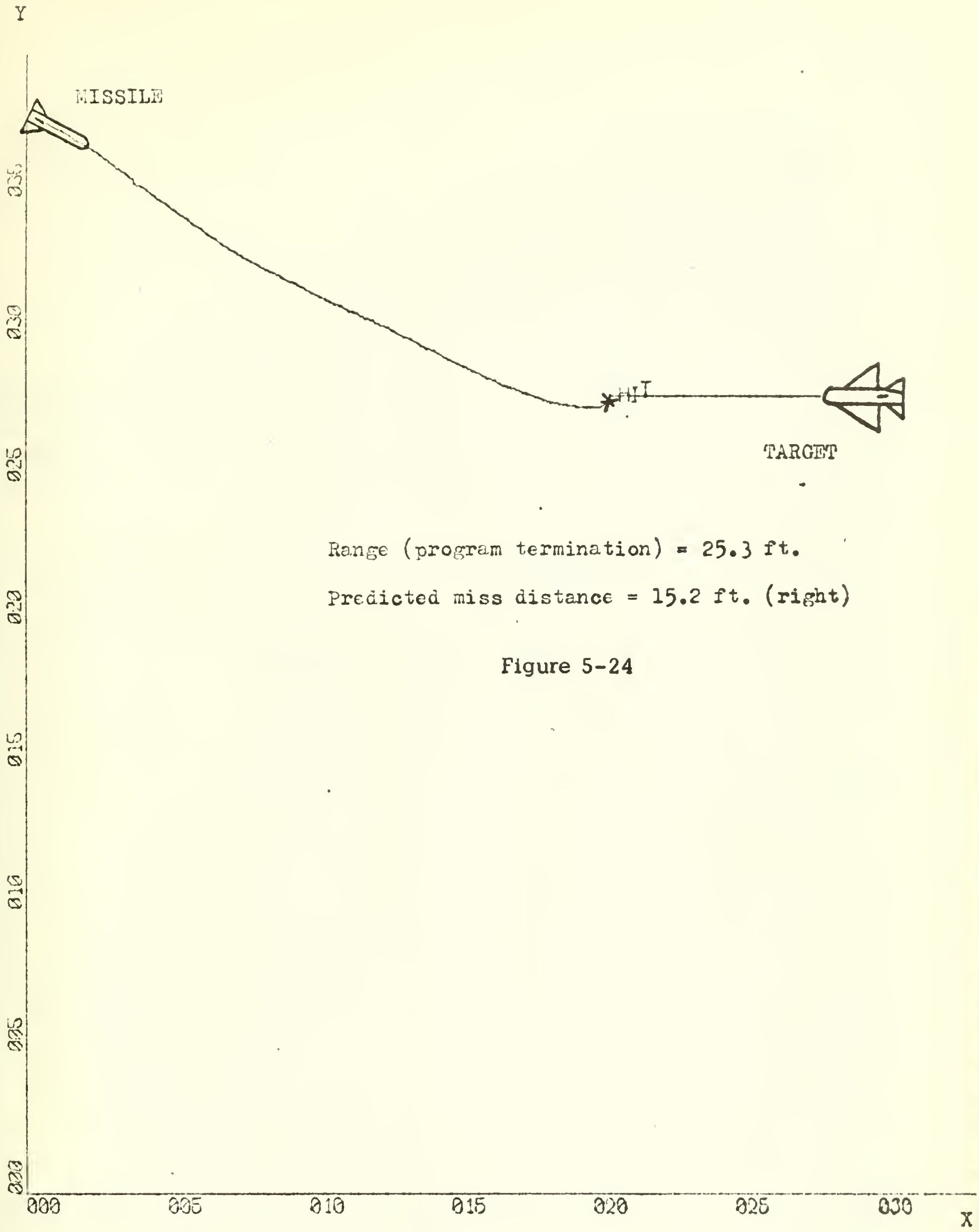


Figure 5-24

(Units = feet)

X-SCALE = 5.00E+03 UNITS/INCH.
Y-SCALE = 5.00E+03 UNITS/INCH.

MISSILE ACTIVE GUIDANCE SIMULATION
D-1-2-0.1-0.1

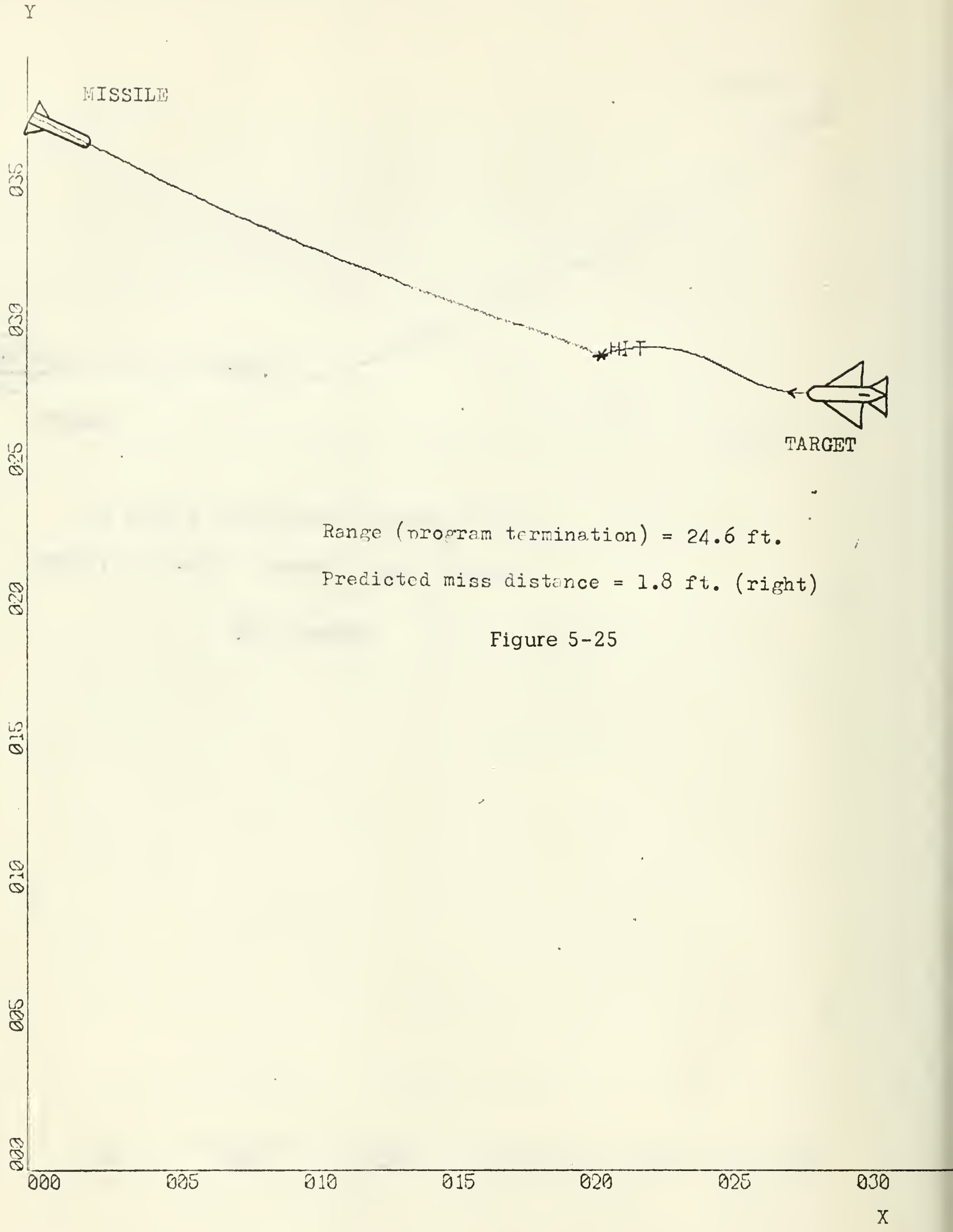


Figure 5-25

X-SCALE = 5.00E+03 UNITS/INCH.
Y-SCALE = 5.00E+03 UNITS/INCH.

(units = feet)

MISSILE ACTIVE GUIDANCE SIMULATION

D-1-3-001-001

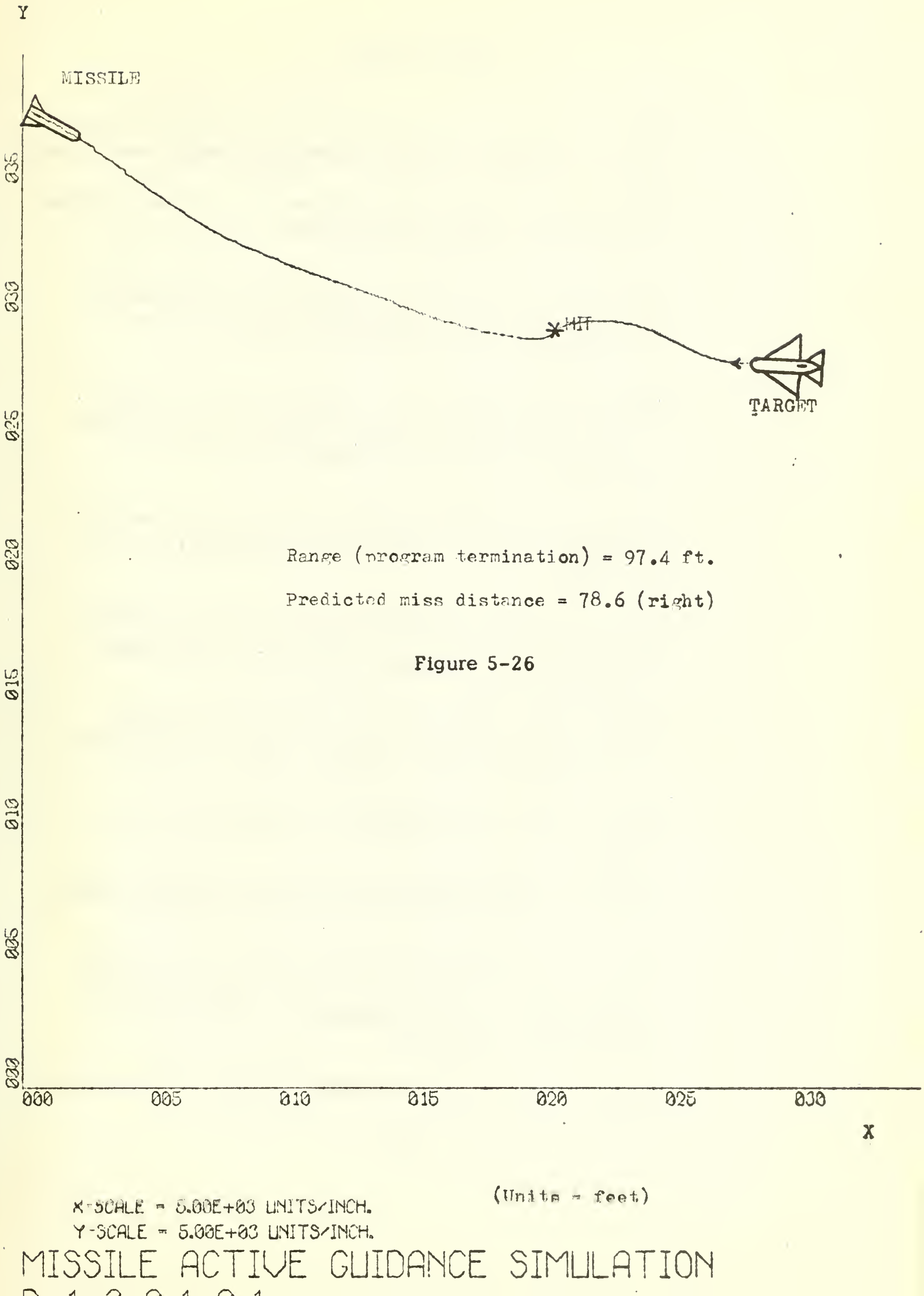

Y
MISSILE
HIT
TARGET
Range (program termination) = 97.4 ft.
Predicted miss distance = 78.6 (right)
035
030
025
020
015
010
005
000
000
005
010
015
020
025
030
X
(Units = feet)
X-SCALE = 5.00E+03 UNITS/INCH.
Y-SCALE = 5.00E+03 UNITS/INCH.
MISSILE ACTIVE GUIDANCE SIMULATION

Figure 5-26

BIBLIOGRAPHY

1. Kalman, R. E. A new approach to linear filtering and prediction problems. Journal of Basic Engineering, March 1960.

2. Kalman, R. E. New methods and results in linear prediction and filtering. RIAS Technical Report, February 1961.

3. Demetry, J. S. and Titus, H. A. Linear control system optimization using a model-based index of performance. U. S. Naval Postgraduate Research Paper, No. 48, November 1964.

4. Demetry, J. S., Strum, R. D. and Titus, H. A. A case study of the discrete optimum filter-controller problem. U. S. Naval Postgraduate Research Paper, No. 57, July 1964.

5. Schmidt, S. F. The application of state space methods to navigation problems. Philco Technical Report, No. 4, July 1964.

6. Ogden, D. J. Optimum digital filter design for sampled data system with illustrative examples. U. S. Naval Postgraduate School Master's Thesis, 1965.

7. Jardine, F. D. Optimal filter design for sampled data system with illustrative examples. U. S. Naval Postgraduate School Master's Thesis, 1965.

8. Lee, R. C. K. Optimal Estimation, Identification and Control. MIT Press, 1964.

9. Locke, A. S. Principles of Guided Missile Design, Guidance. D. Van Nosstrand Company Inc., 1955.

10. Titus, H. A. Computer synthesis of digital and analog filters, predictors, and controllers. Surface Launched Weapon Control Systems Symposium, NOTS, China Lake, Calif., June 1965.

11. Chin, S. S. Missile Configuration Design. McGraw-Hill, 1961.

12. Puckett, A. E. and Ramo, S. Guided Missile Engineering. McGraw-Hill, 1959.

13. Personal conversations with and writings delivered to Professor Titus, U. S. Naval Postgraduate School, 1965-66.

APPENDIX I

THROUGHOUT THIS THESIS COMPUTER PROGRAMS WERE USED, MADE REFERENCE TO OR EXPLAINED. THIS APPENDIX WILL LIST THESE PROGRAMS AND GIVE A BRIEF EXPLANATION OF THE DATA REQUIRED.

PROGRAM FILTER.

THIS PROGRAM CALCULATES THE OPTIMUM STEADY-STATE GAINS G, FOR THE DIGITAL FILTER. THE DATA CARDS ARE EXPLAINED IN THE INITIAL COMMENT CARDS OF THE PROGRAM.

```
      PROGRAM FILTER
C     D1 ORDER OF SYSTEM IN I2 FORMAT
C     D2 SAMPLING INTERVAL IN 8F10.0 FORMAT
C     D3 F MATRIX BY ROWS IN 8F10.0 FORMAT
C     D4 D MATRIX BY COLUMN IN 8F10.0 FORMAT
C     D5  VARIANCE OF EXCITATION NOISE SIGWSQ IN 8F10.6 FORMAT
C     D6 R COVARIANCE MATRIX OF MEASUREMENT NOISE IN 8F10.6 FORMAT
C      PHI SYSTEM TRANSITION MATRIX
C      DEL DISTRIBUTION MATRIX
C      G  OPTIMUM GAIN MATRIX
C      H  OBSERVABLE MATRIX
C      P BEST ESTIMATE OF ERROR COVARIANCE MATRIX
C      Q  EXCITATION NOISE COVARIANCE MATRIX
      DIMENSION P(12,12),Q(12,12),H(12,12),R(12,12),G(12,12),PHIT(12,12
     1,PHI(12,12),DEL(12),DELDELT(12,12),DELS(12,12),DELST(12,12),
     2X(12),XHAT(12),YHAT(12) ,PNEW(12,12)
      READ 33, N
   33 FORMAT (I2)
      READ 2,DT
    2 FORMAT(8F10.0)
      PRINT 1003
 1003 FORMAT(1H1)
      CALL PHIDEL(PHI,DEL,N,DT)
      DO 1001 LL=1,4
      READ 20,SIGWSQ
   20 FORMAT(F10.6)
      DO 1001 LK=1,4
C     INIALIZE MATRICES FOR EACH RUN
      DO 10 I=1,N
      DO 10 J=1,N
      PNEW(I,J)=0.0
      P(I,J)=0.0
      G(I,J)=0.0
      Q(I,J)=0.0
       DELDELT(I,J)=0.0
      DELS(I,J)=0.0
   10 DELST(I,J)=0.0
      READ 2001,R(1,1)
```

PROGRAM FILTER CONTINUED

```
2001 FORMAT(8F10.6)
     DO 30 I=1,N
  30 DELS(I,1)=DEL(I)
     CALL TRANS(DELS,N,N,DELST)
     CALL PROD(DELS,DELST,N,N,N,DELDELT)
     DO 40 I=1,N
     DO 40 J=1,N
  40 Q(I,J)=SIGWSQ*DELDELT(I,J)
     SIGW=SQRTF(SIGWSQ)
     PRINT 1004,SIGW
1004 FORMAT(/ 10X,5HSIGW= /,E20.8)
     P(1,1)=.02
     P(1,2)=0.0
     P(2,1)=0.0
     P(2,2)=1.0
     PRINT 2002,R(1,1),((Q(I,J),J=1,N),I=1,N),((P(I,J),J=1,N),I=1,N)
2002 FORMAT(///20X,8HR(1,1)= ,F10.6/20X,8HQ(I,J)= ,4F10.6/20X,8HP(I,J)=
    1 ,4F10.6)
     H(1,1)=1.
     H(1,2)=0.0
     PRINT 700
 700 FORMAT(//5X,3HG11,7X,3HG12,7X,3HG21,7X,3HG22,7X,3HP11,7X,3HP12,7X,
    1 3HP21,7X,3HP22/)
     DO 1000 KK=1,40
     CALL GP(H,PHI,P,Q,R,2,1,G,PNEW)
     PRINT 800((G(I,J),J=1,N),I=1,N),((PNEW(I,J),J=1,N),I=1,N)
 800 FORMAT (10F10.5)
     DO 11 I=1,N
     DO 11 J=1,N
  11 P(I,J)=PNEW(I,J)
1000 CONTINUE
     PRINT 1005
1005 FORMAT(1H1)
1001 CONTINUE
     END


     SUBROUTINE GP(H,PHI,P,Q,R,KN,KP,G,PNEW)
     DIMENSION H(12,12),PHI(12,12),P(12,12),Q(12,12),R(12,12),G(12,12),
    1PNEW(12,12),HT(12,12),TV1(12,12),TV2(12,12)
     CALL TRANS(H,KP,KN,HT)
     CALL PROD(P,HT,KN,KN,KP,TV1)
     CALL PROD(H,TV1,KP,KN,KP,TV2)
     CALL ADD(TV2,R,KP,KP,TV1)
     CALL RECIP(KP,.000000000001,TV1,TV2,KER)
     IF(KER-2) 101,110,101
 110 PRINT 111
 111 FORMAT(5HKER=2)
 101 CALL PROD(HT,TV2,KN,KP,KP,TV1)
     CALL PROD(P,TV1,KN,KN,KP,G)
     CALL PROD(H,P,KP,KN,KN,TV1)
     CALL PROD(G,TV1,KN,KP,KN,TV2)
```

PROGRAM FILTER CONTINUED

```
      DO 102 I=1,KN
      DO 102 J=1,KN
  102 TV2(I,J)=-TV2(I,J)
      CALL ADD(P,TV2,KN,KN,TV1)
      CALL PROD(PHI,TV1,KN,KN,KN,TV2)
      CALL TRANS(PHI,KN,KN,TV1)
      CALL PROD(TV2,TV1,KN,KN,KN,PNEW)
      CALL ADD(PNEW,Q,KN,KN,PNEW)
      END

      SUBROUTINE TRANS(A,N,M,C)
      DIMENSION A(12,12),C(12,12)
      DO 153 I = 1,N
      DO 153 J=1,M
  153 C(J,I) = A(I,J)
      END

      SUBROUTINE RECIP(N,EP,A,X,KER)
      DIMENSION A(12,12),X(12,12)
      DO 1 I=1,N
      DO 1 J=1,N
    1 X(I,J)=0.0
      DO 2 K=1,N
    2 X(K,K)=1.0
   10 DO 34 L=1,N
      KP=0
      Z=0.0
      DO 12 K=L,N
      IF(Z-ABSF(A(K,L)))11,12,12
   11 Z=ABSF(A(K,L))
      KP = K
   12 CONTINUE
      IF(L-KP)13,20,20
   13 DO 14 J=L,N
      Z=A(L,J)
      A(L,J)=A(KP,J)
   14 A(KP,J)=Z
      DO 15 J=1,N
      Z=X(L,J)
      X(L,J)=X(KP,J)
   15 X(KP,J)=Z
   20 IF(ABSF(A(L,L))-EP)50,50,30
   30 IF(L-N)31,34,34
   31 LP1=L+1
      DO 36 K=LP1,N
      IF(A(K,L))32,36,32
   32 RATIO=A(K,L)/A(L,L)
      DO 33 J=LP1,N
   33 A(K,J)=A(K,J)-RATIO*A(L,J)
      DO 35 J=1,N
   35 X(K,J)=X(K,J)-RATIO*X(L,J)
   36 CONTINUE
```

```
      PROGRAM FILTER CONTINUED

   34 CONTINUE
   40 DO 43 I=1,N
      II=N+1-I
      DO 43 J=1,N
      S=0.0
      IF(II-N)41,43,43
   41 IIP1=II+1
      DO 42 K=IIP1,N
   42 S=S+A(II,K)*X(K,J)
   43 X(II,J)=(X(II,J)-S)/A(II,II)
      KER=1
      RETURN
   50 KER=2
      END


      SUBROUTINE PHIDEL(PHI,DEL,N,DT)
C     VALID ONLY FOR A CONSTANT F MATRIX
      DIMENSION F(12,12),PHI(12,12),TERM(12,12),WORM(12,12)
     1, DEL(12),DELM(12,12),TELM(12,12),DELP(12,12),D(12)
      READ1,((F(IR,IC),IC=1,N),IR=1,N)
    1 FORMAT ((8F10.0))
      READ 1 (D(I),I=1,N)
 1003 PRINT 399,DT,((F(IR,IC),IC=1,N),IR=1,N)
  399 FORMAT(///3HDT=,1F5.3///,7HF(I,J)=/,((8F8.2)))
      PRINT 3991 (D(I),I=1,N)
 3991 FORMAT(////5HD(I)=/(8F8.2))
      NFINAL=1
      TM=0.0
      DO 400 IR=1,N
      DO 400 IC=1,N
      TERM(IR,IC)=0.0
      WORM(IR,IC)=0.0
      TERM(IR,IR)=1.0
      TELM(IR,IC)=TERM(IR,IC)*DT
      DELP(IR,IC)=TELM(IR,IC)
      DELM(IR,IC)=0.0
      DEL(IR)=0.0
  400 PHI(IR,IC)=TERM(IR,IC)
    4 TM=1.0+TM
      DO 500   IR=1,N
      DO 500   IC=1,N
      DO 500   JN=1,N
      DELM(IR,IC)=DELM(IR,IC)-TELM(IR,JN)*F(JN,IC)*DT/(TM+1.0)
  500 WORM(IR,IC)=TERM(IR,JN)*F(JN,IC)*DT/TM+WORM(IR,IC)
      DO 401 IR=1,N
      DO 401 IC=1,N
      TERM(IR,IC)=WORM(IR,IC)
      TELM(IR,IC)=DELM(IR,IC)
      DELP(IR,IC)=DELP(IR,IC)+TELM(IR,IC)
      PHI(IR,IC)=PHI(IR,IC)+TERM(IR,IC)
      DELM(IR,IC)=0.0
  401 WORM(IR,IC)=0.0
```

PROGRAM FILTER CONTINUED

```
      ABC=0.0
      DO 2I=1,N
      DO 2J=1,N
      AA=TERM(I,J)
      AB=ABSF(AA)
      IF(ABC-AB)3,3,2
    3 ABC=AB
    2 CONTINUE
      IF(0.000000005-ABC)5,5,6
    5 GO TO 4
    6 PRINT 502,((PHI(IR,IC),IC=1,N),IR=1,N)
  502 FORMAT(////9X,8HPHI(I,J)///(6E20.8))
      DO 600 I=1,N
      DO 600 K=1,N
      DO 600 J=1,N
  600 DEL(I)=DEL(I)+PHI(I,J)*DELP(J,K)*D(K)
      PRINT 503 (DEL(I),I=1,N)
  503 FORMAT(////9X,6HDEL(I)//(6E20.8)//)
      END


      SUBROUTINE ADD (A,B,N,M,C)
      DIMENSION A(12,12),B(12,12),C(12,12)
      DO 152 I=1,N
      DO 152 J=1,M
  152 C(I,J) = A(I,J) + B(I,J)
      END


      SUBROUTINE PROD (A,B,N,M,L,C)
      DIMENSION A(12,12),B(12,12),C(12,12)
      DO 151 I=1,N
      DO 151 J=1,L
      C(I,J) =0.
      DO 151 K = 1,M
  151 C(I,J) = C(I,J) + A(I,K)*B(K,J)
      END
      END
```

PROGRAM OPTCON

DATA CARDS

D1 CASE THE COMPUTER IS TO EXAMINE.
  CASE 1 Q = I, R = 0 , Q* = 0
  CASE 2 Q = 0, R = 1 , Q* = 0
  CASE 3 Q = I, R = 1, Q* EMPLOYED
D2 N = ORDER OF SYSTEM, NSTAGE = NO. OF ITERATIONS
D3 Q-WEIGHTING MATRIX
D4 F MATRIX
D5 D VECTOR
D6 DT-SAMPLE PERIOD

```
      PROGRAM OPTCON
      DIMENSION PHI(12,12),PSI(12,12),P(12,12),DEL(12),AT(12),
     1GM(12,12),Q(12,12),X(900),ITITLE(12),FM(16),EM(12),HM(12,12),
     1 Y1(900),Y2(900),Y3(900)
C   THIS PROGRAM UTILIZES A COST FUNCTION, J(N)=MINIMUM(SUM X(N)T*Q*X(N)+
C   SUM R*U(N-1)**2).  AN UNLIMITED NUMBER OF ITERATIONS MAY BE MADE AT
C   A COMPUTATION RATE OF 2000 PER MINUTE AFTER THE PROGRAM HAS BEEN
C   COMPILED.  THE OUTPUT OF THIS PROGRAM IS THE FEEDBACK GAIN MATRIX,
C   A TRANSPOSE.  THE FOLLOWING RECURSIVE EQUATIONS WERE DERIVED USING
C   DYNAMIC PROGRAMMING,
C   AT(K)=-(DELT*P(K-1)*PHI)/(DELT*P(K-1)*DEL+R)                       (1)
C   PSI(K)=PHI+DEL*AT(K)                                               (2)
C   P(K)=PSIT(K)*P(K-1)*PSI(K)+Q+R*A(K)*AT(K)                          (3)
C      P(0)=0,AT(0)=0, PSI(0)=0
C   EQUATIONS 1, 2, AND 3 CONSTITUTE THIS PROGRAM
C      CALL IN DATA AND INITIALIZE
      DO 1111 I=1,3
      READ 30,KASE
   30 FORMAT(I1)
      READ 1 ,N,NSTAGE
    1 FORMAT (8I10)
      READ 2,R,DT
    2 FORMAT((4F20.0))
      READ 2 ((Q(I,J),J=1,N),I=1,N)
C     PRINT THE DATA READ IN.
      PRINT 100
  100 FORMAT(1H1)
      PRINT 32,KASE
   32 FORMAT(9X,5HKASE=  ,I3)
      PRINT 3,N,NSTAGE
    3 FORMAT(//9X,2HN=,I3,20X,7HNSTAGE=,I3)
      PRINT 4,R,DT
    4 FORMAT(/9X,2HR=,F15.11,2X,3HDT=,F15.11)
      PRINT 9,((Q(I,J),J=1,N),I=1,N)
    9 FORMAT(/9X,7HQ(I,J)=/(2F15.11))
      CALL PHIDEL(PHI,DEL,N,DT)
      DO 5 I=1,N
      DO 5 J=1,N
      GM(I,J)=0.0
      EM(I)=0.0
      FM(I)=0.0
    5 P(I,J)=Q(I,J)
C     INITIALIZE P(0) FOR CASE TWO
      IF(KASE-2) 35,36,35
   36 P(1,1)=1.0
      P(2,2)=1.0
      P(3,3)=1.0
   35 CONTINUE
      PRINT 19
   19 FORMAT(//2X,6HNSTAGE,4X,7HAT(1,1),3X,7HAT(1,2),4X,6HP(1,1),
     14X,6HP(1,2),4X,6HP(2,1),4X,6HP(2,2))
C     CALCULATE  AT(J)
      DO  22 KK=1,NSTAGE
```

```
      PROGRAM OPTCON CONTINUED

      DEN=0.0
      DO6 I=1,N
      DO6 J=1,N
    6 EM(I)=EM(I)+DEL(J)*P(J,I)
      DO8 I=1,N
      DO7 J=1,N
    7 FM(I)=FM(I)+EM(J)*PHI(J,I)
    8 DEN=DEN+EM(I)*DEL(I)
      DEN=-DEN-R
      DO10I=1,N
      AT(I)=FM(I)/DEN
      FM(I)=0.0
   10 EM(I)=0.0
C     CALCULATE             PSI(K,I,J)
      DO13I=1,N
      DO13J=1,N
   13 PSI(I,J)=PHI(I,J)+DEL(I)*AT(J)
C     CALCULATE             P(K,I,J)
      DO 15 I=1,N
      DO 15 J=1,N
      DO 15 L=1,N
   15 GM(I,J)=GM(I,J)+PSI(L,I)*P(L,J)
      DO 17I=1,N
      DO 17J=1,N
      DO 16 L=1,N
   16 HM(I,J)=HM(I,J)+GM(I,L)*PSI(L,J)

C     CASE 1  TERMINAL CONTROL, OMIT Q(I,J) IN EQUATION FOR P(I,J)
C     CASE2  MINIMUM FUEL       OMIT Q(I,J) IN EQUATION FOR P(I,J)
      IF(KASE-2) 31,31,33
   31 P(I,J)=HM(I,J)+R*AT(I)*AT(J)
      GO TO 17

C     CASE THREE MINIMIZATION OF TIME AND FUEL
   33 P(I,J)=HM(I,J)+Q(I,J)+R*AT(I)*AT(J)
   17 HM(I,J)=0.0
      DO18 I=1,N
      DO18 J=1,N
   18 GM(I,J)=0.0
      PRINT 20,KK,AT(1),AT(2),P(1,1),P(1,2),P(2,1),P(2,2)
   20 FORMAT(I5,(6F10.6))
   22 CONTINUE
 1111 CONTINUE
      END
      SUBROUTINE PHIDEL(PHI,DEL,N,DT)
C     VALID ONLY FOR A CONSTANT F MATRIX
      DIMENSION F(12,12),PHI(12,12),TERM(12,12),WORM(12,12)
     1, DEL(12),DELM(12,12),TELM(12,12),DELP(12,12),D(12)
    1 FORMAT ((8F10.0))
      READ1,((F(IR,IC),IC=1,N),IR=1,N)
      READ 1 (D(I),I=1,N)
 1003 PRINT 399,DT,((F(IR,IC),IC=1,N),IR=1,N)
  399 FORMAT(///3HDT=,1F5.3/  ,7HF(I,J)=/,((2F8.2)))
```

PROGRAM OPTCON CONTINUED

```
      PRINT 3991 (D(I),I=1,N)
 3991 FORMAT(/   5HD(I)=/(2F8.2))
      NFINAL=1
      TM=0.0
      DO 400 IR=1,N
      DO 400 IC=1,N
      TERM(IR,IC)=0.0
      WORM(IR,IC)=0.0
      TERM(IR,IR)=1.0
      TELM(IR,IC)=TERM(IR,IC)*DT
      DELP(IR,IC)=TELM(IR,IC)
      DELM(IR,IC)=0.0
      DEL(IR)=0.0
  400 PHI(IR,IC)=TERM(IR,IC)
    4 TM=1.0+TM
      DO 500   IR=1,N
      DO 500   IC=1,N
      DO 500   JN=1,N
      DELM(IR,IC)=DELM(IR,IC)-TELM(IR,JN)*F(JN,IC)*DT/(TM+1.0)
  500 WORM(IR,IC)=TERM(IR,JN)*F(JN,IC)*DT/TM+WORM(IR,IC)
      DO 401 IR=1,N
      DO 401 IC=1,N
      TERM(IR,IC)=WORM(IR,IC)
      TELM(IR,IC)=DELM(IR,IC)
      DELP(IR,IC)=DELP(IR,IC)+TELM(IR,IC)
      PHI(IR,IC)=PHI(IR,IC)+TERM(IR,IC)
      DELM(IR,IC)=0.0
  401 WORM(IR,IC)=0.0
      ABC=0.0
      DO 2I=1,N
      DO 2J=1,N
      AA=TERM(I,J)
      AB=ABSF(AA)
      IF(ABC-AB)3,3,2
    3 ABC=AB
    2 CONTINUE
      IF(0.000000005-ABC)5,5,6
    5 GO TO 4
    6 PRINT 11,((PHI(I,J),J=1,N),I=1,N)
   11 FORMAT(/9X,9HPHI(I,J)=/(2F15.11))
      DO 600 I=1,N
      DO 600 K=1,N
      DO 600 J=1,N
  600 DEL(I)=DEL(I)+PHI(I,J)*DELP(J,K)*D(K)
      PRINT 12,(DEL(I),I=1,N)
   12 FORMAT(9X,7HDEL(I)=/(1F15.11))
      END
      END
```

```
      PROGRAM MISSILE
C     THIS PROGRAM SIMULATES THE MISSILE CONTROL,DYNAMICS,AND KINEMATI
C     THE  KALMAN FILTER AND OPTIMAL CONTROL IS APPLIED TO THE CONTROL
C     OF THE PLANT.
C     THIS PROGRAM USES VARIABLE FILTER GAINS
C
C     THIS PROGRAM CLOSES THE LOOP ON THE OPTIMUM FILTER-CONTROLLER
C     PROBLEMS. IT ASSUMES THAT STABLE CONTROLLER GAIN MATRIX HAS
C     BEEN COMPUTED ON THE BASIS OF DESIRED RESPONSE AND CALCULATES
C     THE FILTER GAIN MATRIX EACH SAMPLE ON THE BASIS
C     OF THE STATISTICAL PROPERTIES OF THE ANTICIPATED RANDOM DISTURBA
C
C     THE PROGRAM SOLVES THE FOLLOWING EQUATIONS
C          Y(K)=H*X(K)
C          Z(K)=Y(K)+V(K)
C     XS(K)=(I-GH)PHI*XS(K-1)+(I-GH)*DEL*AT(XS(K-1)-DINP(K-1))+G*Z(K)
C     X(K+1)=PHI*X(K)+DEL*W(K)+DEL*AT*(XS(K)-DINP(K))
C     WHERE V IS MEASUREMENT NOISE,W IS THE RANDOM DISTURBANCE,AND
C     DINP IS THE DETERMINISTIC INPUT
C

      DATA CARDS.
         D1 N = ORDER OF SYSTEM, DT= SAMPLE PERIOD
         D2 GAMMA = FLIGHT PATH ANGLE
            SIGMA = LOS ANGLE
            GAMDOT = FLIGHT PATH ANGULAR RATE
            DSIG = LOS ANGULAR RATE

         D3 RLOS = RANGE
            RZ = REQUIRED TARGET RANGE FOR HIT
            XTZ = INITIAL X-COORDINATE OF TARGET
            XTDOT = INITIAL VELOCITY IN X DIRECTION FOR THE TARGET
            YTZ = INITIAL Y-COORDINATE OF TARGET
            YTDOT = INITIAL VELOCITY IN Y DIRECTION FOR THE TARGET
         D4 INITIAL CONDITIONS OF THE STATE VECTOR
         D5 XMZ INITIAL X-COORDINATE OF MISSILE
            YMZ INITIAL Y-COORDINATE OF MISSILE
            VM MISSILE SPEED
         D6 P-BEST ESTIMATE OF ERROR
         D7 F MATRIX
         D8 D VECTOR
         D9 GRAPH TITLE LINE ONE
         D10 AT FEEDBACK GAIN MATRIX FOR CONTROLLER
         D11 GRAPH TITLE LINE TWO
         D12 SIGW = STANDARD DEVIATION OF EXCITATION NOISE
            SIGV = STANDARD DEVIATION OF MEASUREMENT NOISE

      DIMENSION X(12,12),XS(12,12),SIGV(12),Y(12,12),Z(12,12),PHI(12,12
     1,DEL(12,12),H(12,12),AT(12,12),G(12,12),TEMP1(12,12),TEMP2(12,12),
     2TEMP3(12,12),TEMP4(12,12),TELP(12,12),TELP1(12,12),TELP2(12,12),
     3V(12,12),DINP(12,12),IT(12),TME(900),YT(900),XT(900)
     4,XM(900),YM(900),P(12,12),R(12,12)
```

```
      PROGRAM MISSILE CONTINUED

     5,DELTR(12,12),DELS(12),Q(12,12)
     6,SDOT(900),DIFF(900)
C     INPUT SOME CONSTANTS, AS NOTED
      READ 2,N,DT
      READ 1,GAMMA,SIGMA,GAMDOT,DSIG
      READ 1,RLOS,RZ,XTZ,XTDOT,YTZ,YTDOT
      READ 1,(X(I,1),I=1,N)
      READ 1,XMZ,YMZ,VM
    3 FORMAT(6A8)
    2 FORMAT(I5,1F10.0)
    1 FORMAT((8F10.0))
      READ 1,((P(I,J),J=1,N),I=1,N)
      CALL PHIDEL(PHI,DELS,N,DT)
      READ 3,(IT(I),I=1,6)
      READ 1,(AT(1,I),I=1,N)
      READ 3,(IT(I),I=7,12)
      READ 1,SIGW,SIGV(1)
      PRINT 144
  144 FORMAT(4X,8HXMISSILE,8X,8HXTARGET ,8X,8HYMISSILE,8X,8HYTARGET )
      DO 6000 I=1,N
 6000 DEL(I,1)=DELS(I)
      CALL TRANS(DEL,N,N,DELTR)
      CALL PROD(DEL,DELTR,N,N,N,Q)
      DO 200 J=1,N
      DO 200 I=1,N
  200 Q(I,J)=Q(I,J)*SIGW*SIGW
      R(1,1)=SIGV(1)*SIGV(1)
      P(1,1)=R(1,1)
C     KN IS SYSTEM ORDER. KP IS THE NUMBER OF OBSERVABLES.
      KN =N
      KP=1
      H(1,1)=1.0
C     SETTING DINP=0.0 AT THIS POINT INSURES THAT NO DETERMINISTIC
C     INPUTS EXIST PRIOR TO TIME = ZERO
      DO 131  I=1,KN
  131 XS(I,1)=0.0
      NUNIF=1220703125
C     FILTER INITIAL CONDITIONS
      CALL PROD(H,X,KP,KN,KP,Y)
      DO 12 I=1,KP
      CALL RNDEV(NUNIF,DEV)
   12 V(I,1)=SIGV(I)*DEV
      CALL ADD(Y,V,KP,1,Z)
      XS(1,1)=Z(1,1)
      KK=1
      T=0.0
      KK1=0
C     THIS SECTION CALCULATES G, THE GAIN MATRIX
C
 6009 CALL FILTER(KN,KP,PHI,Q,R,H,P,G)
C
C     THIS SECTION CALCULATES XS, THE BEST ESTIMATE
```

PROGRAM MISSILE CONTINUED

```
C       OF THE STATE VECTOR
      CALL PROD(H,X,KP,KN,1,Y)
      DO 10 I=1,KP
      CALL RNDEV(NUNIF,DEV)
   10 V(I,1)=SIGV(I)*DEV
      CALL ADD(Y,V,KP,1,Z)
      CALL PROD(PHI,XS,KN,KN,1,TEMP1)
      CALL PROD(G,H,KN,KP,KN,TEMP2)
      DO 11 I=1,KN
      DO 11 J=1,KN
   11 TEMP2(I,J)=-TEMP2(I,J)
      CALL PROD(TEMP2,TEMP1,KN,KN,1,TEMP3)
      CALL ADD(TEMP1,TEMP3,KN,1,TEMP3)
      CALL ADD(XS,DINP,KN,1,TELP)
      CALL PROD(AT,TELP,1,KN,1,TEMP1)
      CALL PROD(DEL,TEMP1,KN,1,1,TELP)
      CALL PROD(TEMP2,TELP,KN,KN,1,TELP1)
      CALL ADD(TELP,TELP1,KN,1,TELP1)
      CALL PROD(G,Z,KN,KP,1,TELP2)
      CALL ADD(TEMP3,TELP1,KN,1,XS)
      CALL ADD(XS,TELP2,KN,1,XS)

      DINP(1,1)=-DSIG
      DINP(2,1)=-DDSIG
      SDOT(KK)=DSIG
      TME(KK)=T
C
C     THIS SECTION UPDATES THE STATE VECTOR X
C
      CALL RNDEV(NUNIF,DEV)
      W=SIGW*DEV
      CALL PROD(PHI,X,KN,KN,1,TEMP1)
      DO 803 I=1,KN
  803 TEMP2(I,1)=W*DEL(I,1)
      CALL ADD(XS,DINP,KN,1,TELP)
      CALL PROD(AT,TELP,1,KN,1,TELP1)
      CALL PROD(DEL,TELP1,KN,1,1,TELP2)
      CALL ADD(TEMP1,TEMP2,KN,1,X)
      CALL ADD(X,TELP2,KN,1,X)
C
C     THIS SECTION DETERMINES THE EFFECT OF THE PLANT
C     ON THE KINEMATICS
      DKSIG=DSIG
      DIFF(KK)=X(1,1)-XS(1,1)
      GAMDOT=X(1,1)
      GDOT(KK)=GAMDOT
      GAMMA =GAMMA+GAMDOT*DT
      XMDOT=VM*COSF(GAMMA)
  133 YMDOT=VM*SINF(GAMMA)
  134 YRDOT =YTDOT-YMDOT
      XRDOT =XTDOT-XMDOT
      RDOT=((YRDOT*SINF(SIGMA))+(XRDOT*COSF(SIGMA)))
      RDSIG=((YRDOT*COSF(SIGMA))-(XRDOT*SINF(SIGMA)))
```

```
      RLOS =RLOS+RDOT*DT
      DSIG=RDSIG/RLOS
      SIGMA =SIGMA+DSIG*DT
      DDSIG=(DSIG-DKSIG)/DT
      XMZ=XMZ+XMDOT*DT
      YMZ = YMZ+YMDOT*DT
      XM(KK)=XMZ
      YM(KK)=YMZ
      XTZ =XTZ+XTDOT*DT
      YTZ =YTZ+YTDOT*DT
      YT(KK) =YTZ
      XT(KK) =XTZ
      RL(KK)=RLOS
      PRINT 140,KK
      PRINT 141,XM(KK),XT(KK),YM(KK),YT(KK)
  140 FORMAT(2X,I5)
  141 FORMAT(2X,4E16.7)
      KK=KK+1
      T = T + DT
      PRINT 190, DSIG,SIGMA
  190 FORMAT(6H DSIG=,1E16.7,2X,7H SIGMA=,1E16.7)
      IF(RDOT)146,145,145
  145 PRINT 135,RDOT
      GOTO 6013
  146 IF(KK-900 )6012,6012,6013
 6012 IF(RLOS-100.)6013,6013,6011
 6011 GOTO 6009
 6013 PRINT 136, RLOS
  135 FORMAT(6H RDOT=,1E16.7)
  136 FORMAT(6H RLOS=,1E16.7)
      A=ATANF(RDSIG/RDOT)
      TMISS=((RLOS*RDSIG)/RDOT)*COSF(A)
      PRINT 137,TMISS
  137 FORMAT(7H TMISS=,1E16.7)
      KK=KK-1
      MC =1
      LA=4H  MI
      CALL DRAW(KK,XM,YM,MC,0,LA,IT,0,0,0,0,0,0,7,8,0,LAST)
      MC=3
      LA=4H   T
      CALL DRAW(KK,XT,YT,MC,0,LA,IT,0,0,0,0,0,0,7,8,0,LAST)
      END
      SUBROUTINE PROD (A,B,N,M,L,C)
      DIMENSION A(12,12),B(12,12),C(12,12)
      DO 151 I=1,N
      DO 151 J=1,L
      C(I,J)=0.
      DO 151 K=1,M
  151 C(I,J)=C(I,J)+A(I,K)*B(K,J)
      END
      SUBROUTINE ADD (A,B,N,M,C)
      DIMENSION A(12,12),B(12,12),C(12,12)
      DO 152 J=1,M
```

```
      PROGRAM MISSILE CONTINUED

      DO 152 I=1,N
  152 C(I,J)=A(I,J)+B(I,J)
      END


      SUBROUTINE FILTER(N,KP,PHI,Q,R,H,P,G)
C      PHI SYSTEM TRANSITION MATRIX
C      DEL DISTRIBUTION MATRIX
C      G  OPTIMUM GAIN MATRIX
C      H  OBSERVABLE MATRIX
C      P BEST ESTIMATE OF ERROR COVARIANCE MATRIX
C      Q  EXCITATION NOISE COVARIANCE MATRIX
      DIMENSION P(12,12),Q(12,12),H(12,12),R(12,12),G(12,12),PHIT(12,12
     1,PHI(12,12),DEL(12),DELDELT(12,12),DELS(12,12),DELST(12,12),
     2PNEW(12,12)
      CALL GP(H,PHI,P,Q,R,N,KP,G,PNEW)
      DO 11 I=1,N
      DO 11 J=1,N
   11 P(I,J)=PNEW(I,J)
      END

      SUBROUTINE GP(H,PHI,P,Q,R,KN,KP,G,PNEW)
      DIMENSION H(12,12),PHI(12,12),P(12,12),Q(12,12),R(12,12),G(12,12)
     1PNEW(12,12),HT(12,12),TV1(12,12),TV2(12,12)
      CALL TRANS(H,KP,KN,HT)
      CALL PROD(P,HT,KN,KN,KP,TV1)
      CALL PROD(H,TV1,KP,KN,KP,TV2)
      CALL ADD(TV2,R,KP,KP,TV1)
      CALL RECIP(KP,.0000000000001,TV1,TV2,KER)
      IF(KER-2) 101,110,101
  110 PRINT 111
  111 FORMAT(5HKER=2)
  101 CALL PROD(HT,TV2,KN,KP,KP,TV1)
      CALL PROD(P,TV1,KN,KN,KP,G)
      CALL PROD(H,P,KP,KN,KN,TV1)
      CALL PROD(G,TV1,KN,KP,KN,TV2)
      DO 102 I=1,KN
      DO 102 J=1,KN
  102 TV2(I,J)=-TV2(I,J)
      CALL ADD(P,TV2,KN,KN,TV1)
      CALL PROD(PHI,TV1,KN,KN,KN,TV2)
      CALL TRANS(PHI,KN,KN,TV1)
      CALL PROD(TV2,TV1,KN,KN,KN,PNEW)
      CALL ADD(PNEW,Q,KN,KN,PNEW)
      END

      SUBROUTINE TRANS(A,N,M,C)
      DIMENSION A(12,12),C(12,12)
      DO 153 I = 1,N
      DO 153 J=1,M
  153 C(J,I) = A(I,J)
      END
```

PROGRAM MISSILE CONTINUED

```
      SUBROUTINE TRANS(A,N,M,C)
      DIMENSION A(12,12),C(12,12)
      DO 153 I = 1,N
      DO 153 J=1,M
  153 C(J,I) = A(I,J)
      END

      SUBROUTINE RECIP(N,EP,A,X,KER)
      DIMENSION A(12,12),X(12,12)
      DO 1 I=1,N
      DO 1 J=1,N
    1 X(I,J)=0.0
      DO 2 K=1,N
    2 X(K,K)=1.0
   10 DO 34 L=1,N
      KP=0
      Z=0.0
      DO 12 K=L,N
      IF(Z-ABSF(A(K,L)))11,12,12
   11 Z=ABSF(A(K,L))
      KP = K
   12 CONTINUE
      IF(L-KP)13,20,20
   13 DO 14 J=L,N
      Z=A(L,J)
      A(L,J)=A(KP,J)
   14 A(KP,J)=Z
      DO 15 J=1,N
      Z=X(L,J)
      X(L,J)=X(KP,J)
   15 X(KP,J)=Z
   20 IF(ABSF(A(L,L))-EP)50,50,30
   30 IF(L-N)31,34,34
   31 LP1=L+1
      DO 36 K=LP1,N
      IF(A(K,L))32,36,32
   32 RATIO=A(K,L)/A(L,L)
      DO 33 J=LP1,N
   33 A(K,J)=A(K,J)-RATIO*A(L,J)
      DO 35 J=1,N
   35 X(K,J)=X(K,J)-RATIO*X(L,J)
   36 CONTINUE
   34 CONTINUE
   40 DO 43 I=1,N
      II=N+1-I
      DO 43 J=1,N
      S=0.0
      IF(II-N)41,43,43
   41 IIP1=II+1
      DO 42 K=IIP1,N
   42 S=S+A(II,K)*X(K,J)
   43 X(II,J)=(X(II,J)-S)/A(II,II)
      KER=1
```

```
      RETURN
   50 KER=2
      END
      SUBROUTINE PHIDEL (PHI,DEL,N,DT)
      DIMENSION F(12,12),PHI(12,12),TERM(12,12),WORM(12,12)
     1, DEL(12),DELM(12,12),TELM(12,12),DELP(12,12),D(12)
C     VALID ONLY FOR A CONSTANT F MATRIX
C     DT= SAMPLING INTERVAL
      NFINAL=1
      READ 1((F(IR,IC),IC=1,N),IR=1,N)
      READ 1 (D(I),I=1,N)
    1 FORMAT ((8F10.0))
      TM=0.0
 1003 PRINT 399,DT,((F(IR,IC),IC=1,N),IR=1,N)
  399 FORMAT(///4H DT=, 1F5.3////,8H F(I,J)=/,4(4E16.7/))
      PRINT 3991 (D(I),I=1,N)
 3991 FORMAT(///6H D(I)=/,4(1E16.7/))
      DO 400 IR=1,N
      DO 400 IC=1,N
      TERM(IR,IC)=0.0
      WORM(IR,IC)=0.0
      TERM(IR,IR)=1.0
      TELM(IR,IC)=TERM(IR,IC)*DT
      DELP(IR,IC)=TELM(IR,IC)
      DELM(IR,IC)=0.0
      DEL(IR)=0.0
  400 PHI(IR,IC)=TERM(IR,IC)
    4 TM=1.0+TM
      DO 500   IR=1,N
      DO 500   IC=1,N
      DO 500   JN=1,N
      DELM(IR,IC)=DELM(IR,IC)-TELM(IR,JN)*F(JN,IC)*DT/(TM+1.0)
  500 WORM(IR,IC)=TERM(IR,JN)*F(JN,IC)*DT/TM+WORM(IR,IC)
      DO 401 IR=1,N
      DO 401 IC=1,N
      TERM(IR,IC)=WORM(IR,IC)
      TELM(IR,IC)=DELM(IR,IC)
      DELP(IR,IC)=DELP(IR,IC)+TELM(IR,IC)
      PHI(IR,IC)=PHI(IR,IC)+TERM(IR,IC)
      DELM(IR,IC)=0.0
  401 WORM(IR,IC)=0.0
      ABC=0.0
      DO 2I=1,N
      DO 2J=1,N
      AA=TERM(I,J)
      AB=ABSF(AA)
      IF(ABC-AB)3,3,2
    3 ABC=AB
    2 CONTINUE
      IF(0.000000005-ABC)5,5,6
    5 GO TO 4
    6 PRINT 502,((PHI(IR,IC),IC=1,N),IR=1,N)
  502 FORMAT(//9X,8HPHI(I,J)///((4E16.7)))
```

PROGRAM MISSILE CONTINUED

```
      DO 600 I=1,N
      DO 600 K=1,N
      DO 600 J=1,N
  600 DFL(I)=DEL(I)+PHI(I,J)*DELP(J,K)*D(K)
      PRINT 503 (DEL(I),I=1,N)
  503 FORMAT(///9X,6HDEL(I)//(8F15.9)//)
      RETURN
      END
      END
```

PROGRAM MISSILE CONTINUED

```
C     EVASION
      YTDOT =YTDOT+10.
C      EVASION LEFT-RIGHT-LEFT
      IF(KK-30)320,321,321
  320 YTDOT=YTDOT+20.
      GOTO 323
  321 YTDOT=YTDOT-20.
  323 CONTINUE
      IF(YTZ-5000.)324,324,325
  324 YTDOT=YTDOT+30.
      IF (YTZ-2000.)326,326,325
  326 YTDOT=0.0
  325 CONTINUE
C      END OF EVASION
```

APPENDIX II

The missile guidance simulation is a transfer function relating (γ) and (σ). Since the objective of this study is to examine the effect of a digital filter, an arbitrary transfer function was selected which could represent any present day missile. This transfer function contains a time delay due to the radar, a filtering action due to the control (hydraulics etc.), and a second order system representing the missile dynamics.

$$\sigma/\gamma = \frac{A}{S + T_R} \cdot \frac{B}{S + T_c} \cdot \frac{C}{S^2 + DS + E} \qquad \text{(A-1)}$$

As mentioned above the intention here is to demonstrate the capability of the digital filter in the presence of a large magnitude of noise. The values of the gains and time constants were selected for equation (A-1) after examining several different missile systems which use proportional navigation. Refer to [13] for further elaboration on determining the F and D matrices for a particular system.

INITIAL DISTRIBUTION

		No. Copies
1.	Defense Documentation Center Cameron Station Alexandria, Virginia 22314	20
2.	Library U. S. Naval Postgraduate School Monterey, California 93940	2
3.	Professor H. A. Titus (Thesis Advisor) Electronics Department U. S. Naval Postgraduate School Monterey, California 93940	4
4.	Lt. Harry A. Steele, Jr. c/o Professor H. A. Titus Electronics Department U. S. Naval Postgraduate School Monterey, California 93940	2
5.	U. S. Naval A. R. Development Center (Attn: Max Fernboch) Johnsville, Pennsylvania 18974	2
6.	Bureau of Naval Weapons Navy Department Washington 25, D. C.	1
7.	Hughes Aircraft Co. (Attn: D. K. Richardson) Culver City, California	1

Unclassified

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)

U.S. Naval Postgraduate School
Monterey, California 93940

2a. REPORT SECURITY CLASSIFICATION

Unclassified

2b. GROUP

3. REPORT TITLE

A Study of Digital Control Design Methods

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Masters' Thesis in Electrical Engineering, May, 1966

5. AUTHOR(S) (Last name, first name, initial)

Steele, Jr., Harry A. Lieutenant U.S. Navy

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
May 1966	92	13

8a. CONTRACT OR GRANT NO.

b. PROJECT NO.

c.

d.

9a. ORIGINATOR'S REPORT NUMBER(S)

9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

#2 [illegible] 1/27/71

10. AVAILABILITY/LIMITATION NOTICES

~~Qualified requestors may obtain copies of this report from DDC.~~

11. SUPPLEMENTARY NOTES

12. SPONSORING MILITARY ACTIVITY

Bureau of Naval Weapons

13. ABSTRACT

The development of microelectronics has brought into being large capacity digital memories in a small package. In the foreseeable future even more advances can be seen in this trend. Therefore the use of digital computers in control systems will play an even larger role than today.

This work involves a fourth order system to simulate the control and dynamics of a missile. Proportional navigation is used as the guidance method. Studied are the effects of applying different controls which are considered best from a computer study and the effects of applying digital filtering methods. Although these studies were applied to a specific problem, an attempt is made to keep the discussion general in order that the methods may be considered for other problems.

DD FORM 1 JAN 64 1473

Unclassified

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Missile Digital simulation Digital filter Proportional navigation						

INSTRUCTIONS

1. ORIGINATING ACTIVITY: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization *(corporate author)* issuing the report.

2*a*. REPORT SECURITY CLASSIFICATION: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2*b*. GROUP: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. REPORT TITLE: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. DESCRIPTIVE NOTES: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. AUTHOR(S): Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. REPORT DATE: Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7*a*. TOTAL NUMBER OF PAGES: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7*b*. NUMBER OF REFERENCES: Enter the total number of references cited in the report.

8*a*. CONTRACT OR GRANT NUMBER: If appropriate, enter the applicable number of the contract or grant under which the report was written.

8*b*, 8*c*, & 8*d*. PROJECT NUMBER: Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9*a*. ORIGINATOR'S REPORT NUMBER(S): Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9*b*. OTHER REPORT NUMBER(S): If the report has been assigned any other report numbers *(either by the originator or by the sponsor)*, also enter this number(s).

10. AVAILABILITY/LIMITATION NOTICES: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through
________________________________."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through
________________________________."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through
________________________________."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. SUPPLEMENTARY NOTES: Use for additional explanatory notes.

12. SPONSORING MILITARY ACTIVITY: Enter the name of the departmental project office or laboratory sponsoring *(paying for)* the research and development. Include address.

13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS)*, *(S)*, *(C)*, or *(U)*.

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, roles, and weights is optional.